Fitting unintegrated PDFs for LHC

EINN2009, Milos, Greece

Albert Knutsson, DESY

Collaborators: A. Bacchetta, H. Jung, K. Kutak

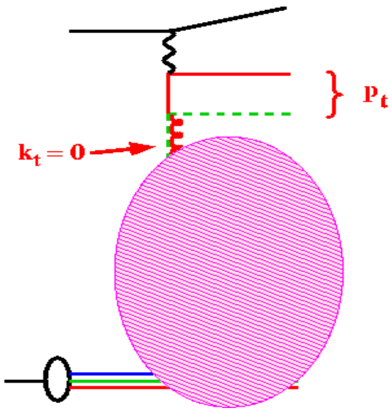
Outline

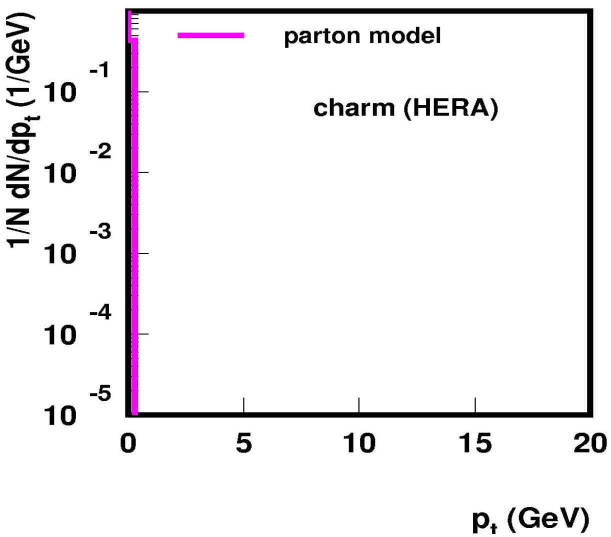
- **Introduction – unintegrated PDFs and Monte Carlo**
- **Determining the unintegrated PDFs**
    - **the fitting procedure**
    - **the x dependence**
    - **the k_t-dependence**
- **Summary**

Introduction

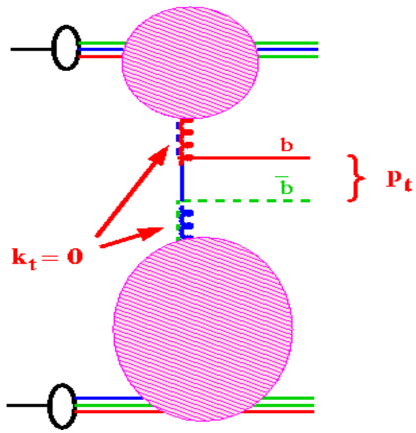
J. Collins, H. Jung hep-ph/0508280

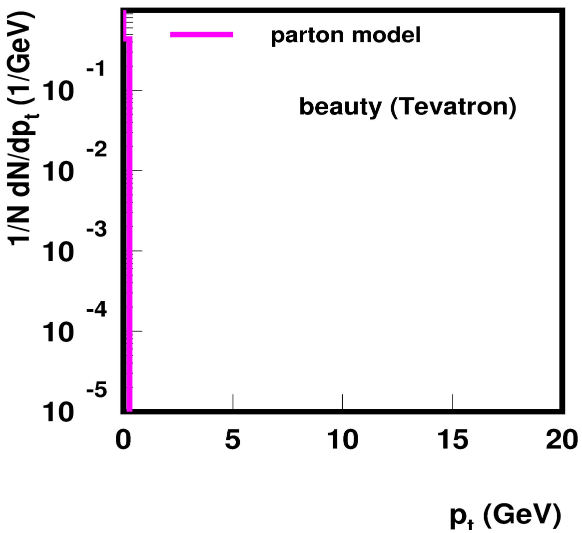
Heavy quark production at HERA



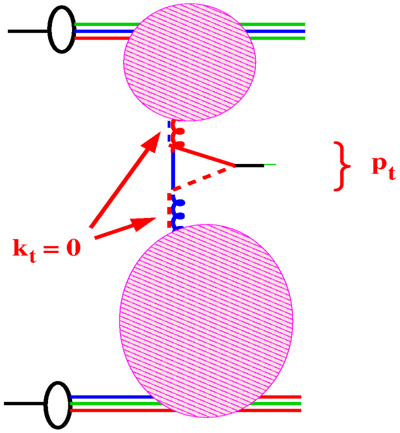


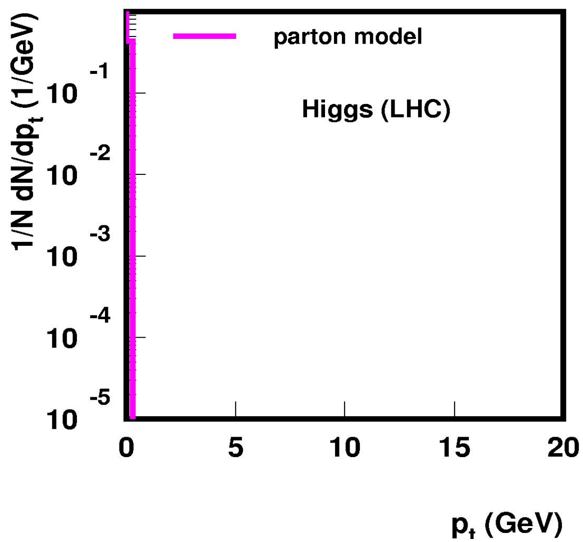
Heavy quark production in pp collisions





Higgs production in pp collisions





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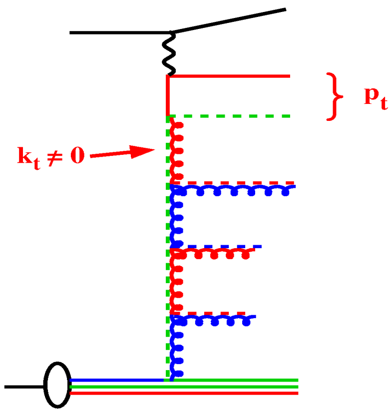

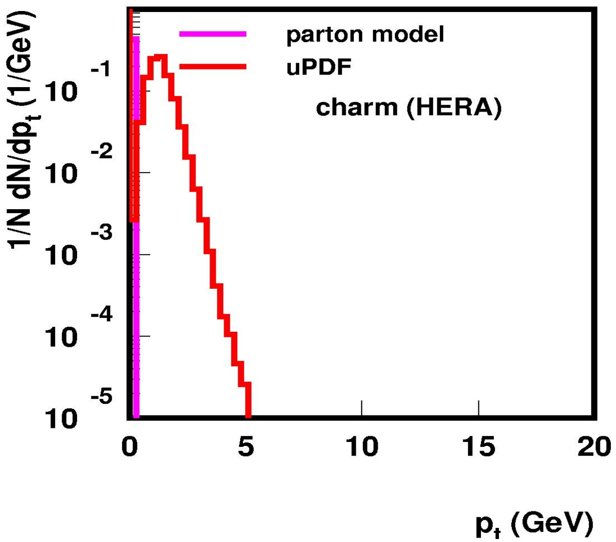

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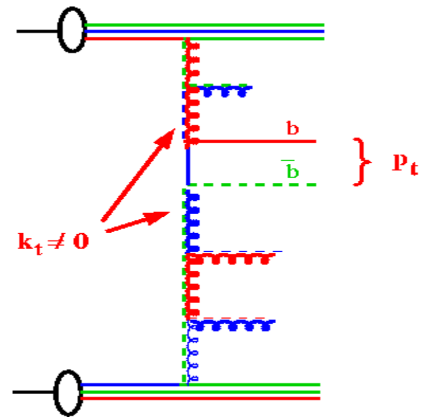

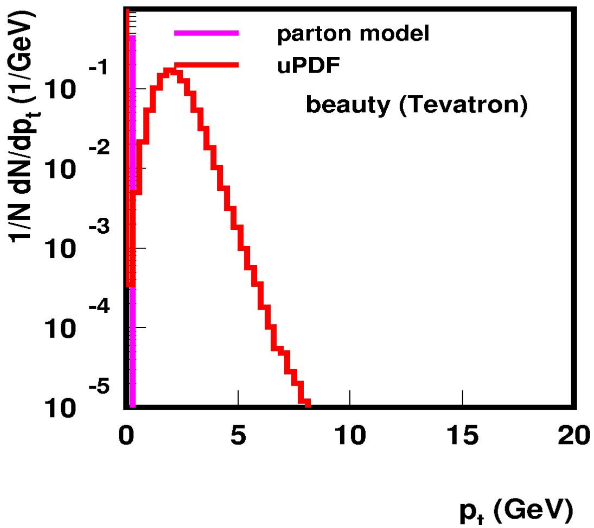

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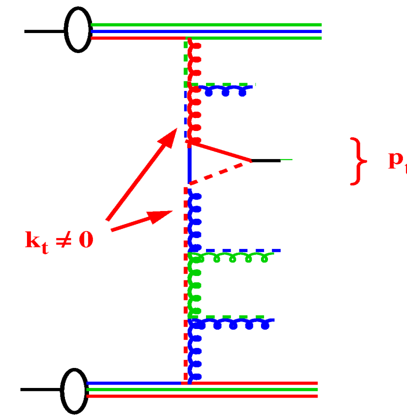
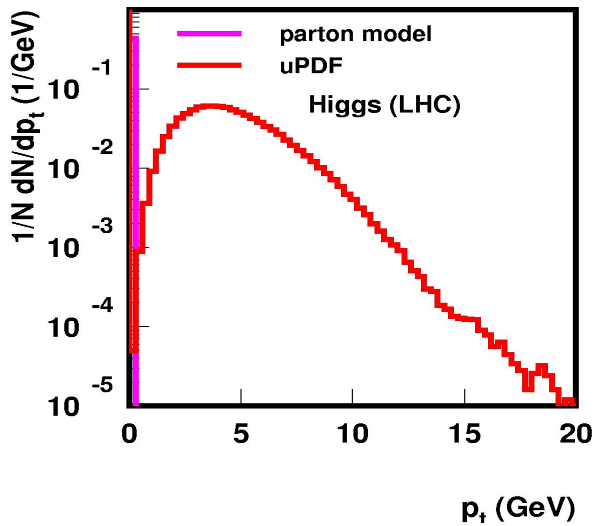




→ **Need k_t-dependence. Kinematics correctly treated by using unintegrated PDFs. Gives significant transverse momentum of in the final state.**

Different approaches

- Take derivative from PDF:

$$f(x, k_\perp^2) \sim \frac{d\Delta xg(x, k_\perp^2)_{\mathrm{DGLAP}}}{d\log k_\perp^2}$$

- The KMR approach. Use normal PDFs. Let the last emission generate transverse momentum via the Sudakov form factor.

→ **Can be used in the DGLAP approach, where a strong ordering in k_t is assumed.**

In this talk:

•**CCFM (Catani Ciafaloni Fiorani Marchesini) approach.**

Parton evolution with angular ordering instead of strong ordering in k_t.

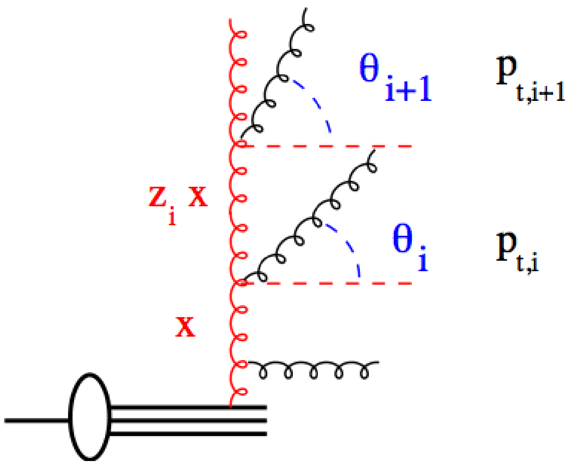
Use a unintegrated PDF with parameters determined by fits to data.

CCFM and CASCADE MC generator

- **uPDF starting disitribution (example):**

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T - \mu)^2}{2\sigma^2})$$

- **Defined at some starting scale and evolved to higher scales by emissions of gluons according to the *CCFM* evolution scheme.**
  Angular ordering of emitted gluon (Color coherence). No explicit k_t-ordering.



- **CCFM is usually refered to as the bridge between DGLAP and BFKL.**

- **CCFM and uPDFs are fully implemented in the general purpose ep/pp MC generator CASCADE (H. Jung, *Comput.Phys.Commun.143:100-111,2002*).**

Determining the uPDF parameters

Determination of the parameters in the gluon density

Work done together with A. Bacchetta, H. Jung, K. Kutak

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T - \mu)^2}{2\sigma^2})$$

N: **Normalization**
B: **Small x behaviour**
***C*=4: Large x behaviour (Roughly fixed by momentum sum rules.)**
μ, σ: **Determines the shape of the intrinsic k_T of the gluon**

The parameters N,B,C, μ, σ, are not theoretically calculable.

→ **We need to fit the uPDF to experimental data.**

Determine the uPDF by fits to data

1. Calculate cross-section using Monte Carlo for a given set of parameter values
2. Compare to data, calculate Chi2 and feed it to MINUIT
3. MINUIT (e.g. the MIGRAD method) estimates new parameter values
4. Iterate 1. - 3. until Chi2 is minimized

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This means that if MINUIT needs 100 iterations to minimize Chi2, the generator is run 100 times, not simultaneously:

If one MC generator run takes 1 hour (understatement)**, the minimization takes 100hours.**

One may need exclusive measurements

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A lot of MC statistics. Minimization >> 100h.

Also delicate: Fitting several "event types" simultaneously,
e.g. Charm production and inclusive jet production
Above method makes separated event generation difficult.

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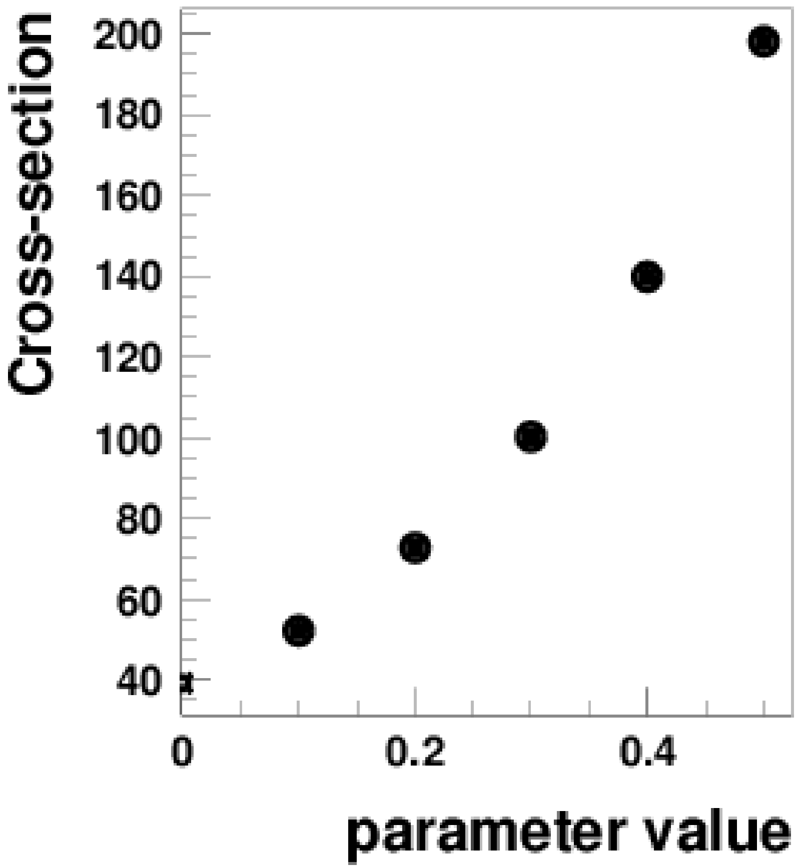
New Approach: Describe **parameter dependence before parameter fitting**,
by using a ***grid in parameter space***.

1-dim Example

Simplest possible example

1 parameter, 1 data cross-section

1. Build up the grid



Monte Carlos cross-sections

Simple Example

Simplest possible example

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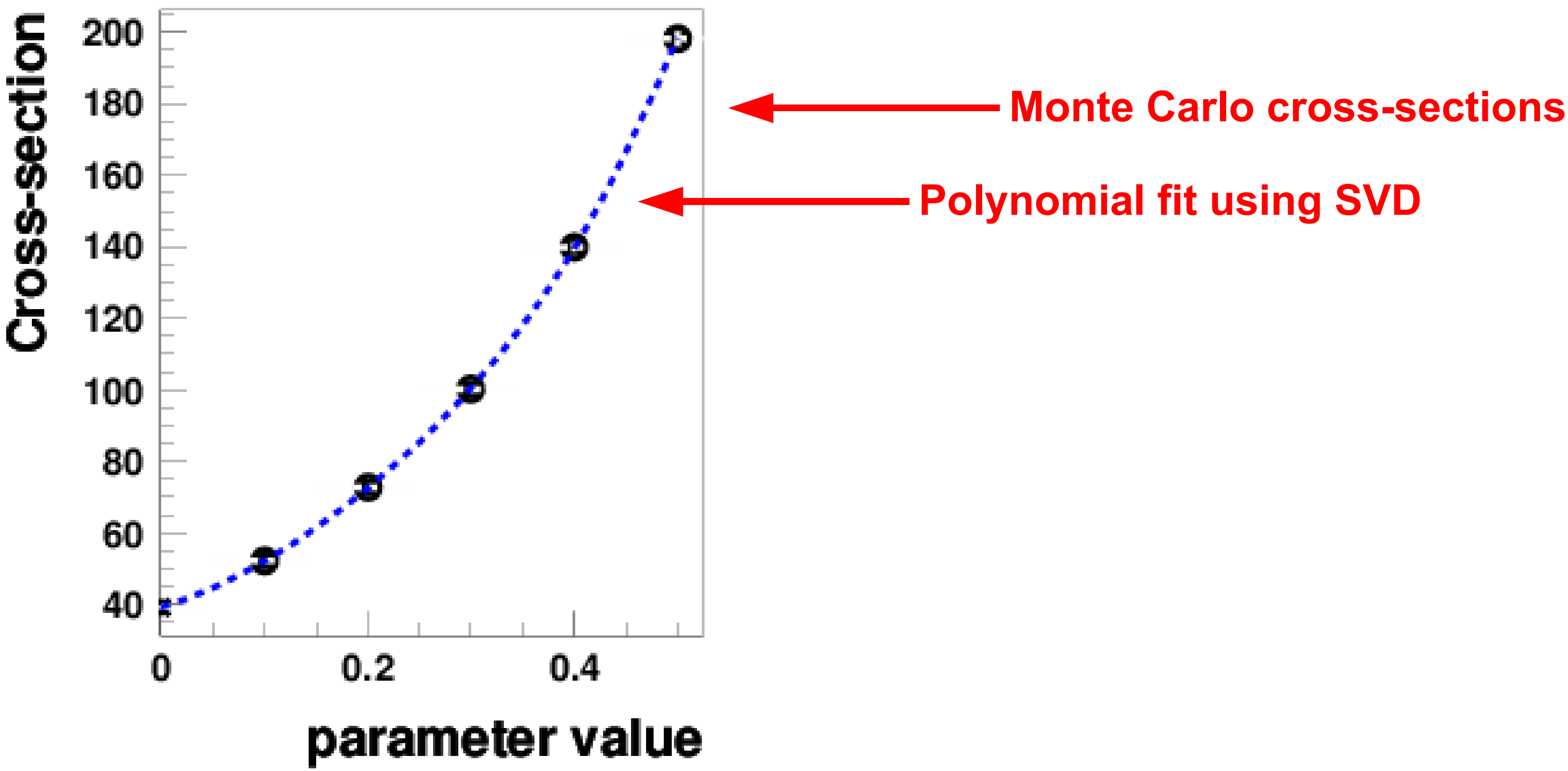
2. Determine polynomial using SVD



Simple Example

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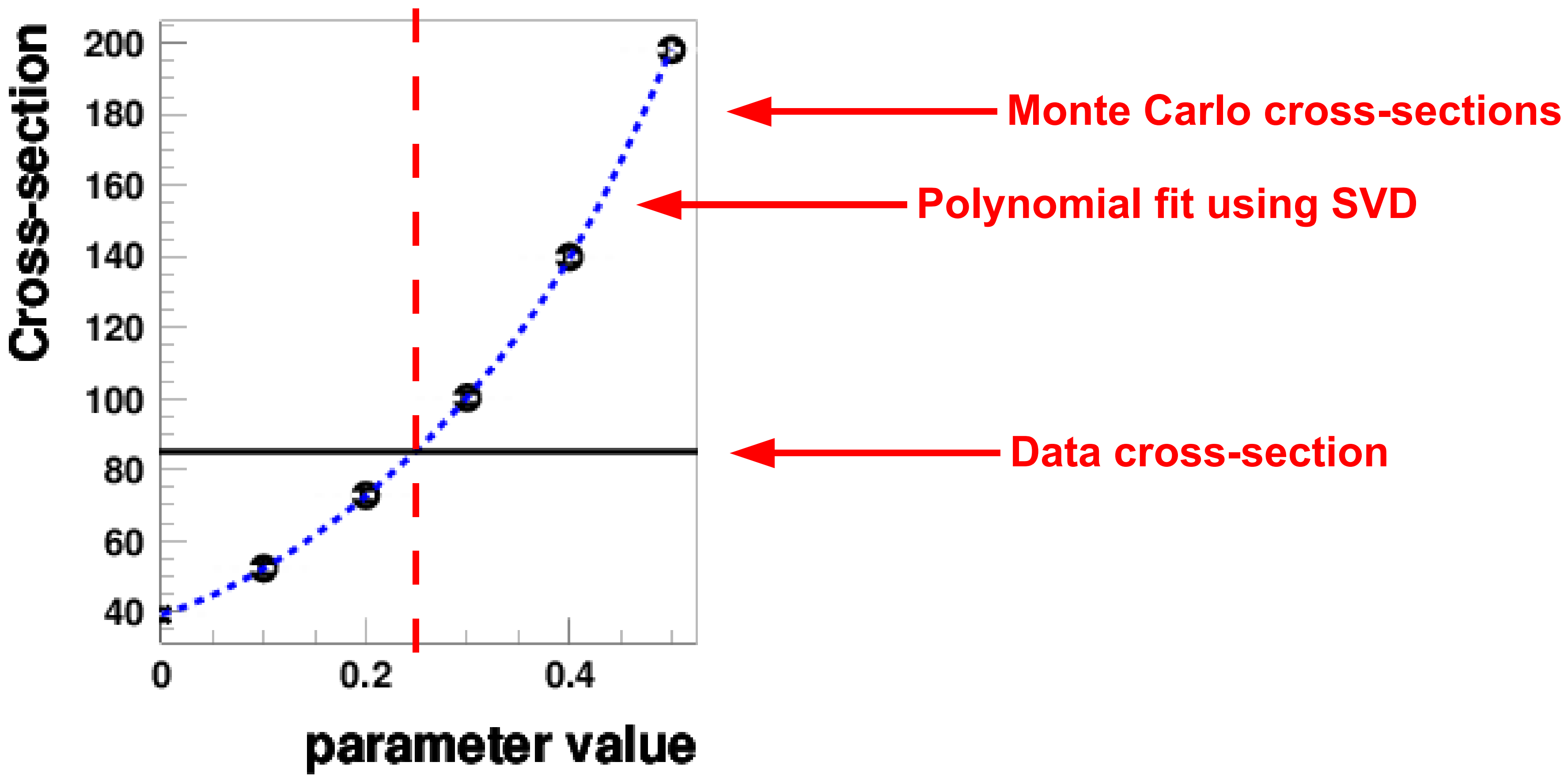
1 parameter, 1 data cross-section

3. Minimize Chi2 to data



New fitting approach

1. Build up a grid in parameter – cross section space using Monte Carlo.

If you have a CPU farm (or use the *GRID*) this ultimately takes the time of running the MC generator once.

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2. Fit polynomials to the Monte Carlo grid.

$$\sigma_{\text{poly}} = A + \sum_{1}^{N} B_i \cdot p_i + \sum_{1}^{N} C_i \cdot p_i^2 + \sum_{i=1}^{N} \sum_{j=i+1}^{N} D_{ij} \cdot p_i p_j + H.O.$$

A, B, C **and** D **are determined by fitting the polynomial to the parameter grid by Singular Value Decomposition.**

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Takes care of correlation between parameters

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3. Determine PDF parameters, p_i **, by fitting all the polynomials to data simultaneously**

Also this takes only a few seconds.

Step 3. is done by Chi2-minimization using MINUIT.

Determination of the parameters in the gluon density

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T - \mu)^2}{2\sigma^2})$$

Used in the CASCADE MC generator:
Evolved according to the CCFM equation – parton showers – (hadronization)

- First goal determine the x-dependence: **Normalization (N) and the small x behaviour (B)**

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Used in the CASCADE MC generator:
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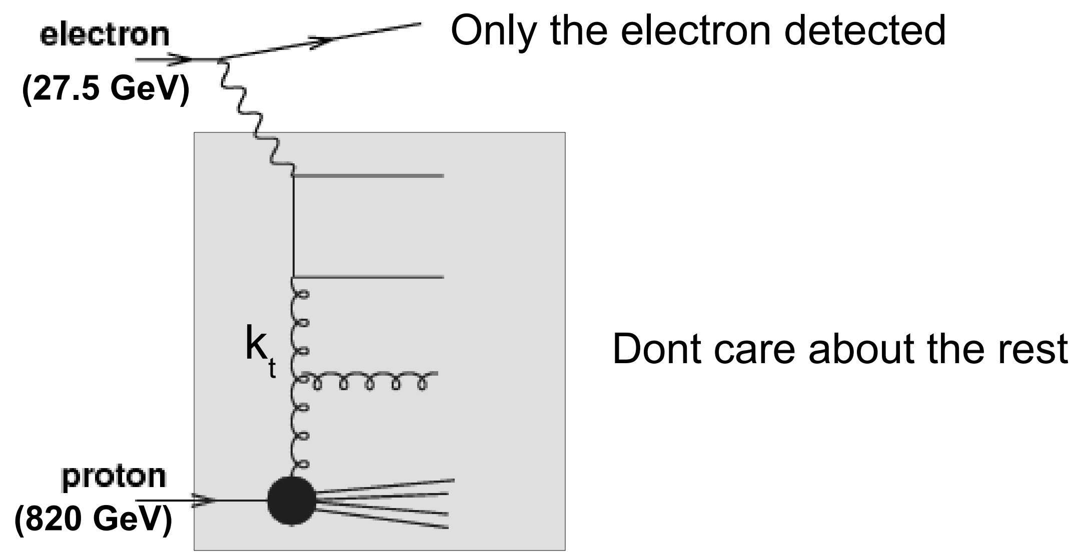
•First goal determine the x-dependence: **Normalization (N) and the small x behaviour (B)**

•Use the proton structure function, F2. The data covers a large range in *x*

$3 \cdot 10^{-5} < x < 2 \cdot 10^{-1}$
$1 < Q^2 < 150$ GeV^2

....but should be fairly insensitive to the kt-dependent part of the gluon. Inclusive measurement with no restrictions on the hadronic final state.



Fitting the x dependence to the proton structure - F2

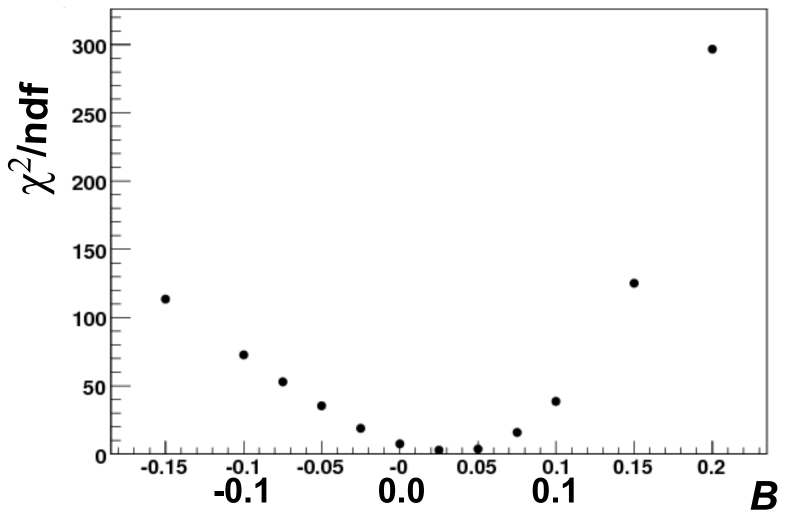
$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T-\mu)^2}{2\sigma^2})$$

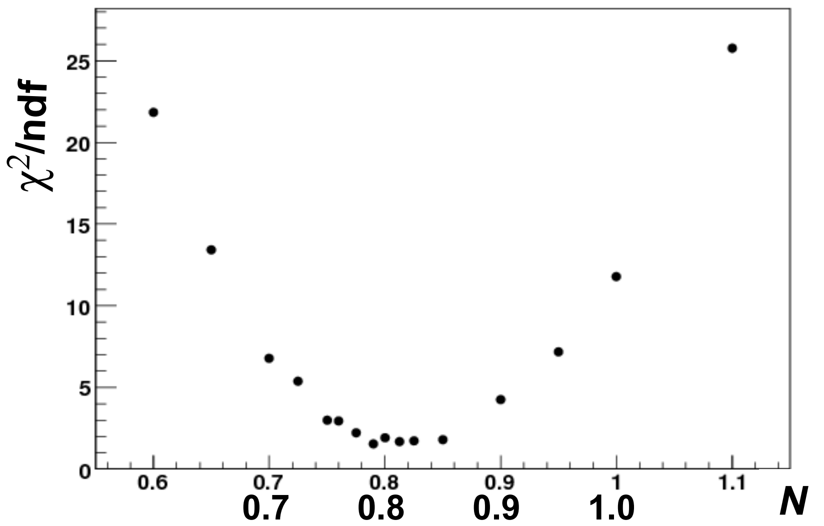
Fitting F2 in the range x < 0.005, Q2>4.5, gives:

<u>Minimum</u>
N = 0.807 +/- 0.016
B = 0.029 +/- 0.004
Chi2/ndf=1.2

This is a good fit which reconstructs the parameter values in an already existing PDF tuned to F2 within the same kinematic range... (Good validation of the method.)

χ^2 - profiles as for the fitted parameters:





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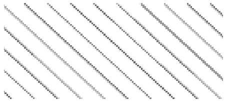
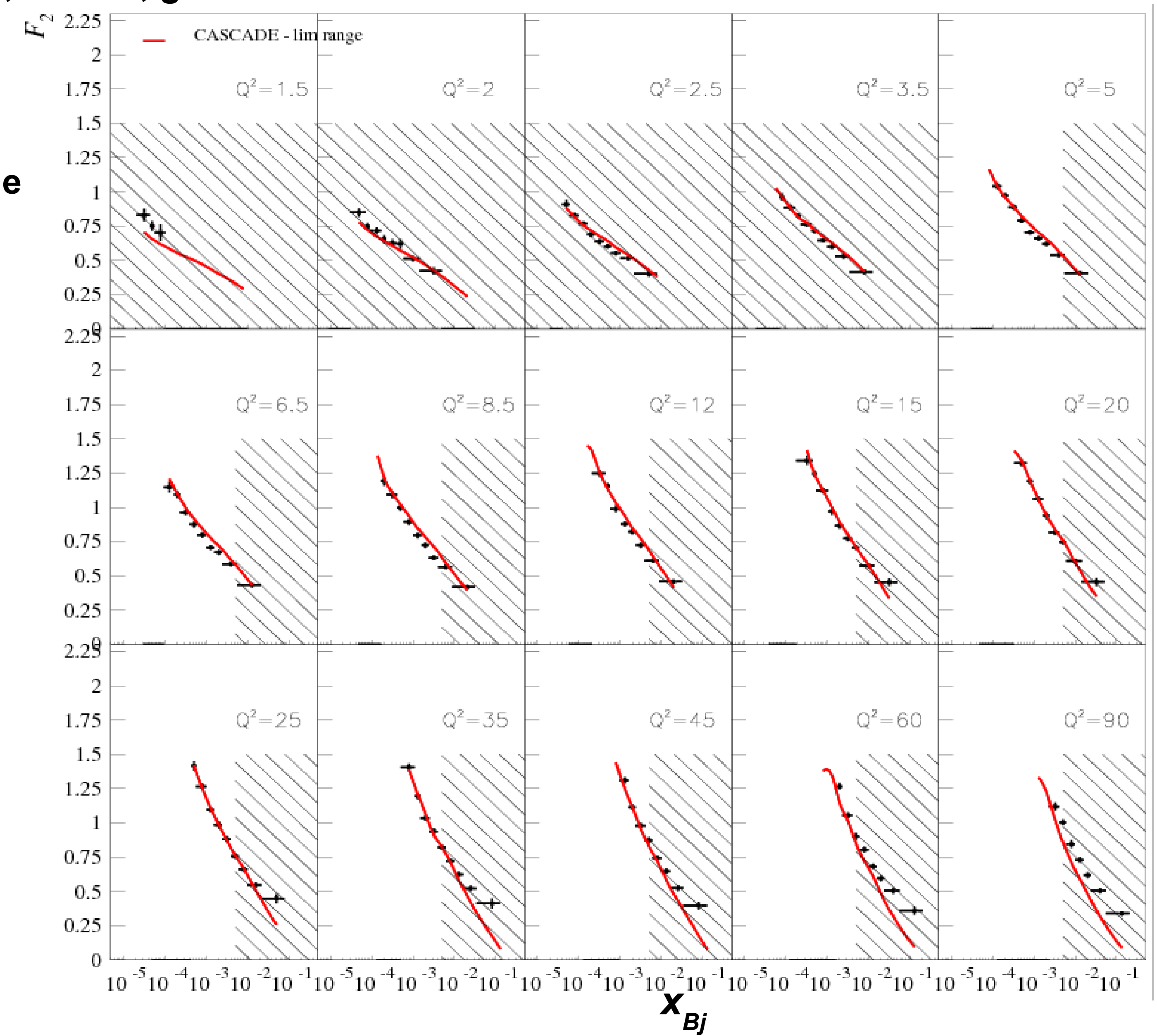
 = not fitted

Bad description of data outside the fitted range.



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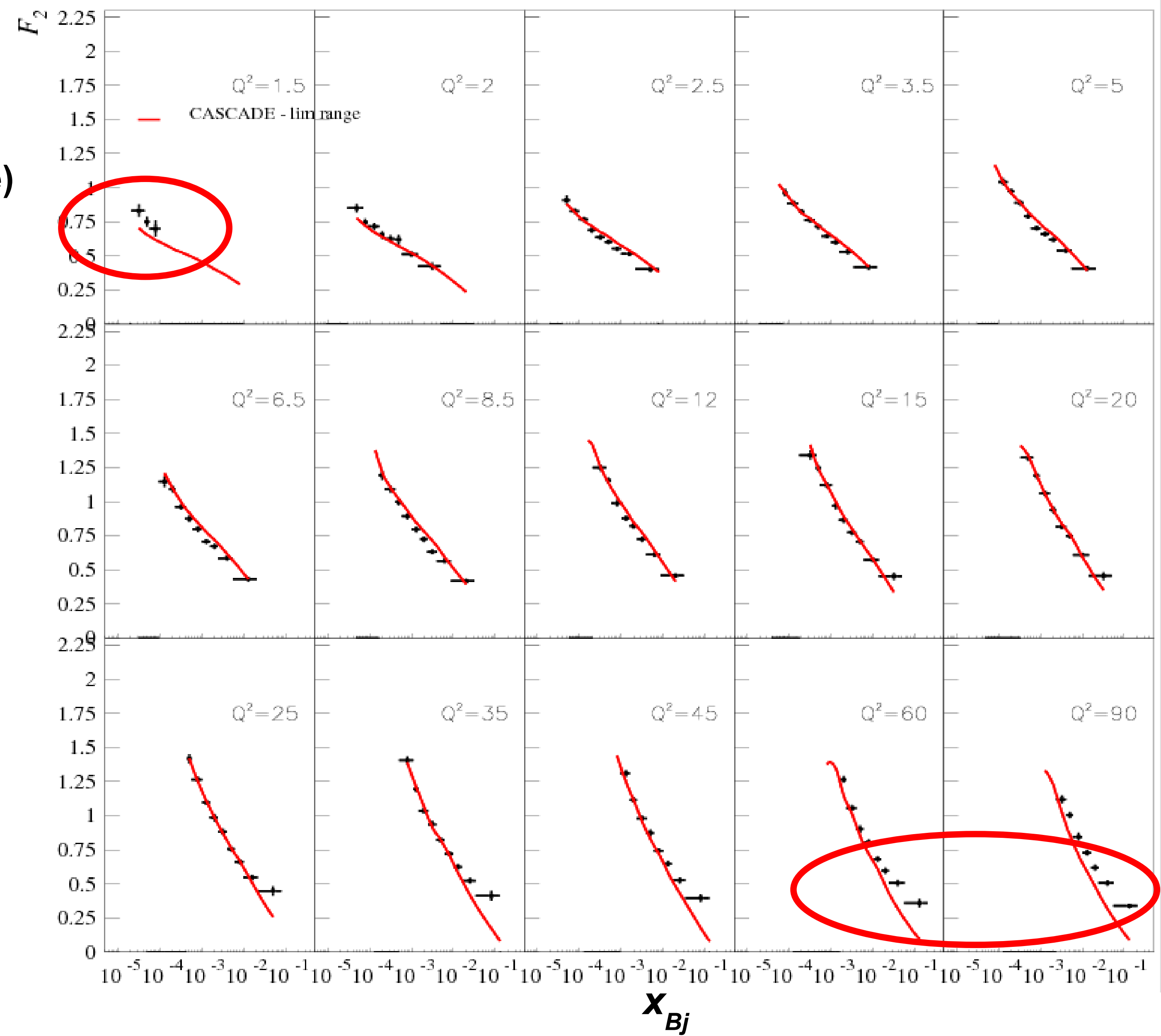


Fitting the x dependence to the proton structure - F2

However if we open up the phase space and fit all F2 data points, we obtain the minimum:

<u>Minimum</u>

N = 0.767 +/- 0.001

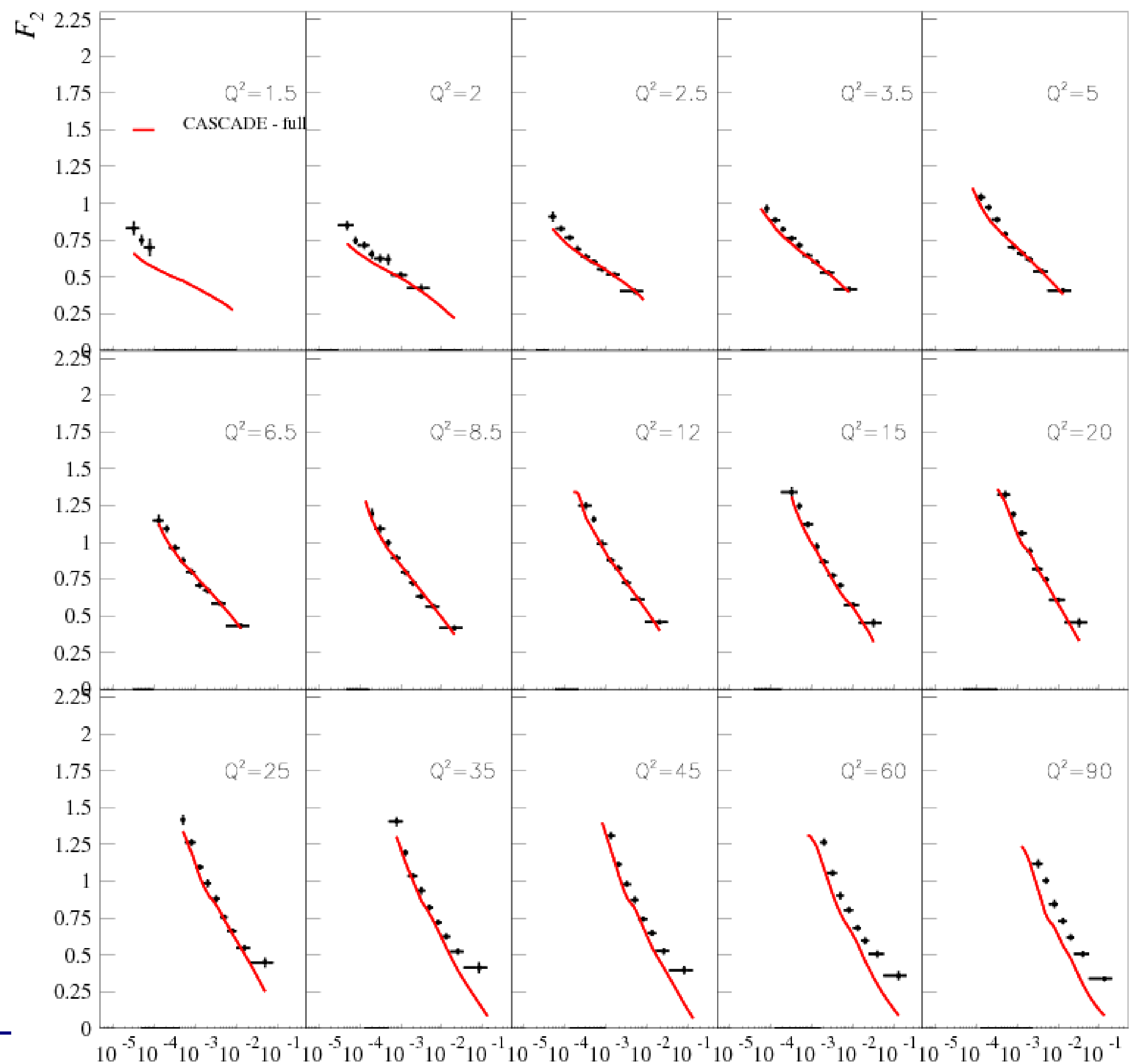
B = 0.028 +/- 0.000

Chi2/ndf = 5.4

Essentially the same minimum

Data at high and low x are still not described.

Suggests that we need more freedom in the fit needed.



Changing the parameterization of the uPDF

Inspired by the CTEQ group we added an extra factor in the PDF parameterization:

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot (1 - Dx) \cdot G(k_T)$$

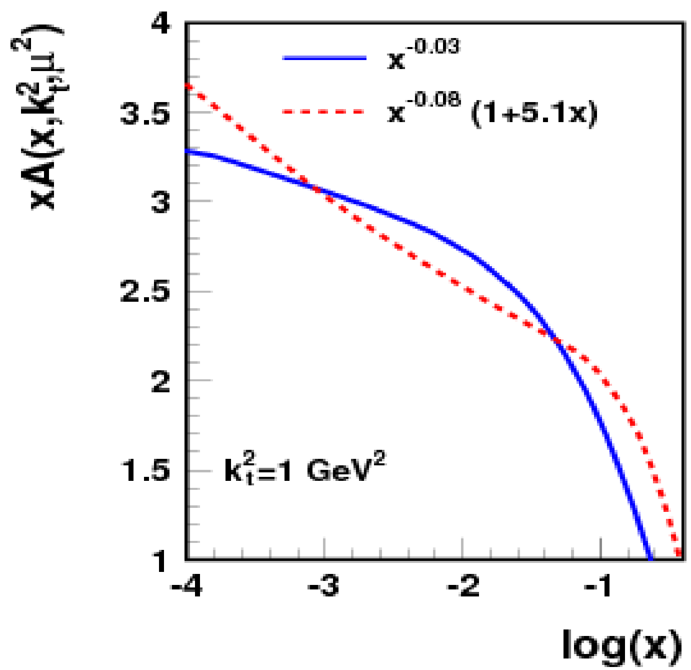
...and performed a 3 dimensional fit of N,B and D.

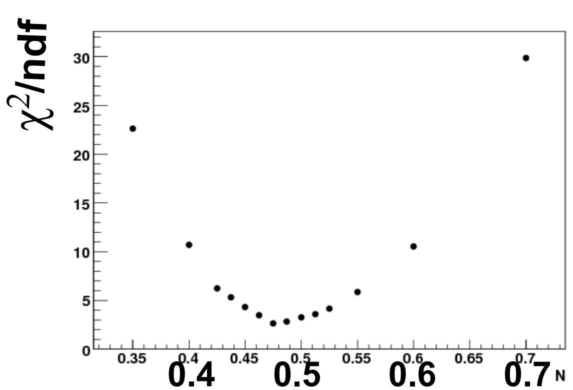
Fitting the x dependence to the proton structure - F2

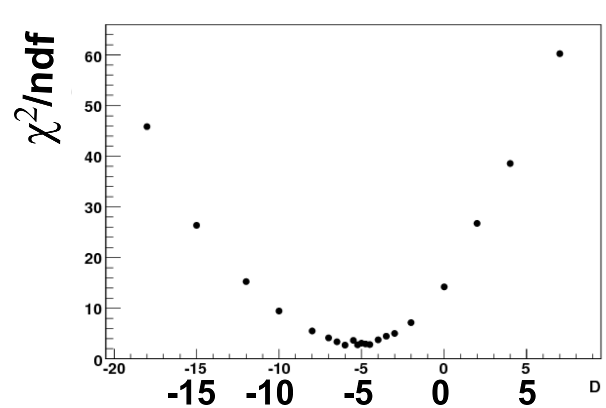
$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot (1 - Dx) \cdot G(k_T)$$

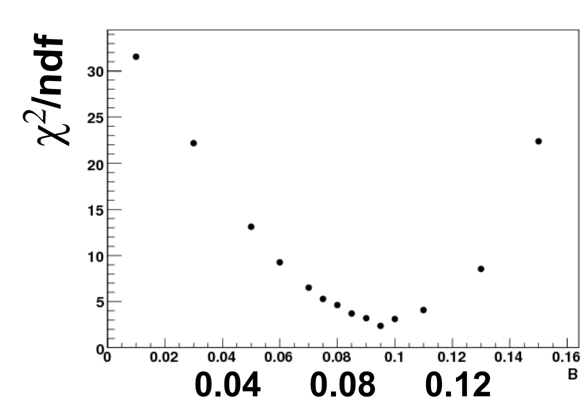
Fitting F2 over the full range in x gives a slightly different gluon then before.
uPDF allowed to be more pronounced at low and high x:

Minimum
N = 0.487 +/- 0.007
B = 0.097 +/- 0.003
D = -5.10 +/- 0.35
Chi2/ndf = 2.8
(Before: Chi2/ndf = 5.4)









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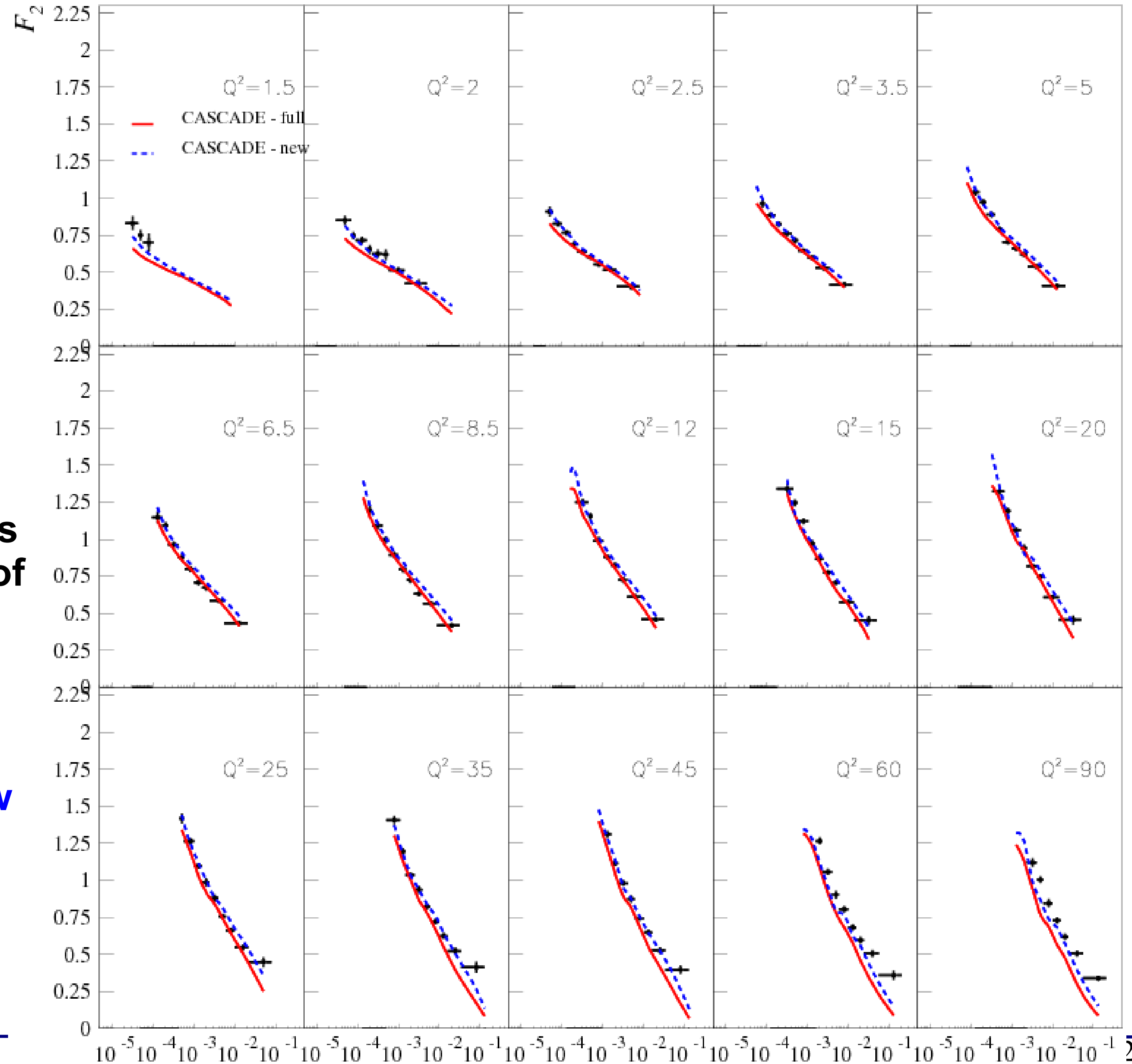
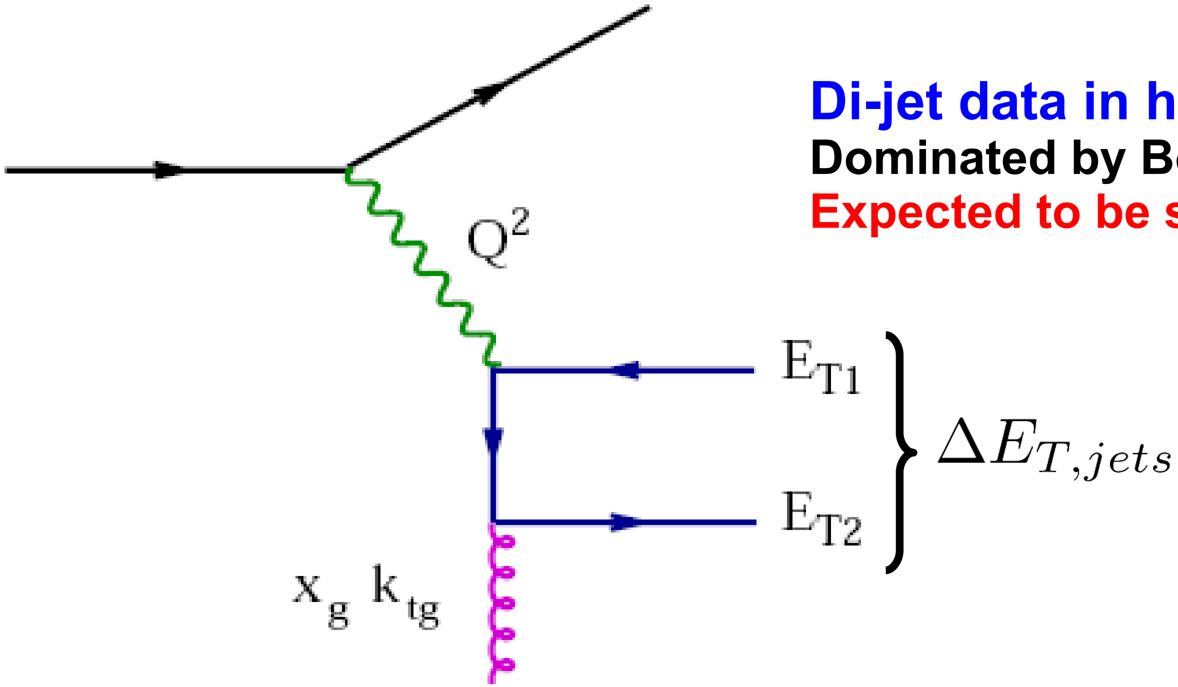


The extra factor (1-Dx) gives a significant improvement of the data description at low and high x.

Blue dashed line is the new fit

Fitting the k_t dependence of the uPDF

kt-sensitive data



Di-jet data in high energy ep-collisions.
Dominated by Boson Gluon Fusion.
Expected to be sensitive to the gluon.

Integrated PDF: DGLAP

LO: **Gluon collinear with proton**

$$k_{t,\mathrm{gluon}} = 0$$

$$\Delta E_{T,jets} = 0 \text{ in } \textit{HCM}$$

Higher orders:

$$k_{t,\mathrm{gluon}} \neq 0$$

$$\Delta E_{T,\mathrm{jets}} \neq 0$$

Unintegrated PDF: CCFM or BFKL

$$k_{t,\mathrm{gluon}} \neq 0$$

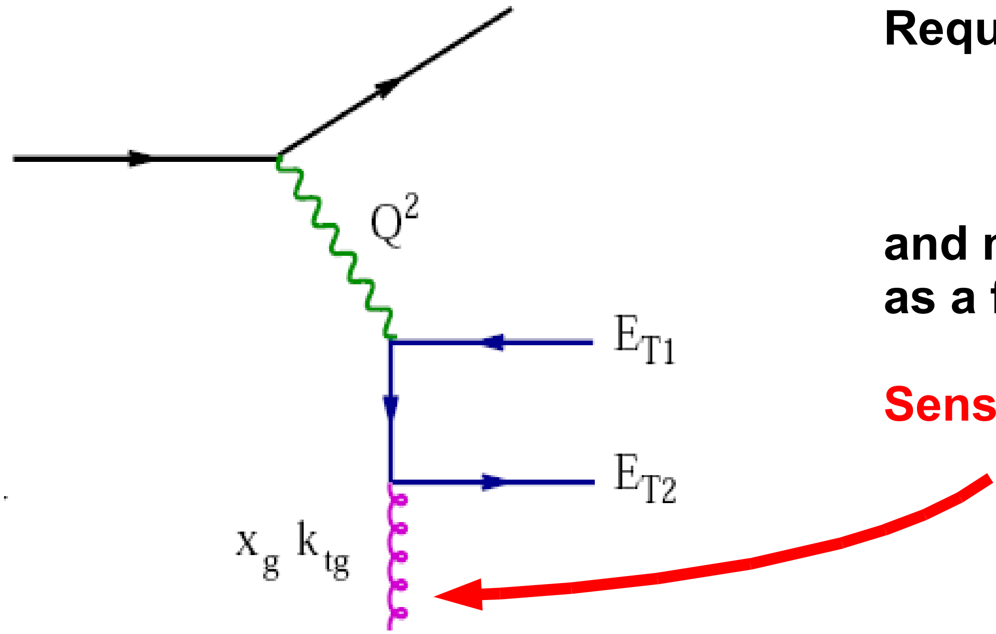
$$\Delta E_{t,\mathrm{jets}} \neq 0$$

already at LO

k_t-sensitive data

Fit unintegrated gluon density to *HERA di-jet data*

Target hard di-jets.
Dominated by BGF, sensitivity to gluon.



Require $E_{T,\text{ jet } 1} > (5+\Delta)\ \text{GeV}$
$E_{T,\text{ jet } 2} > 5\ \text{GeV}$

and measure jet cross-section as a function of Δ

Sensitivity to gluon k_t

Data from H1 Collab., A. Aktas et al., Eur. Phys. J. C33 (2004) 477
Inclusive Dijet Production at Low x_Bj in DIS

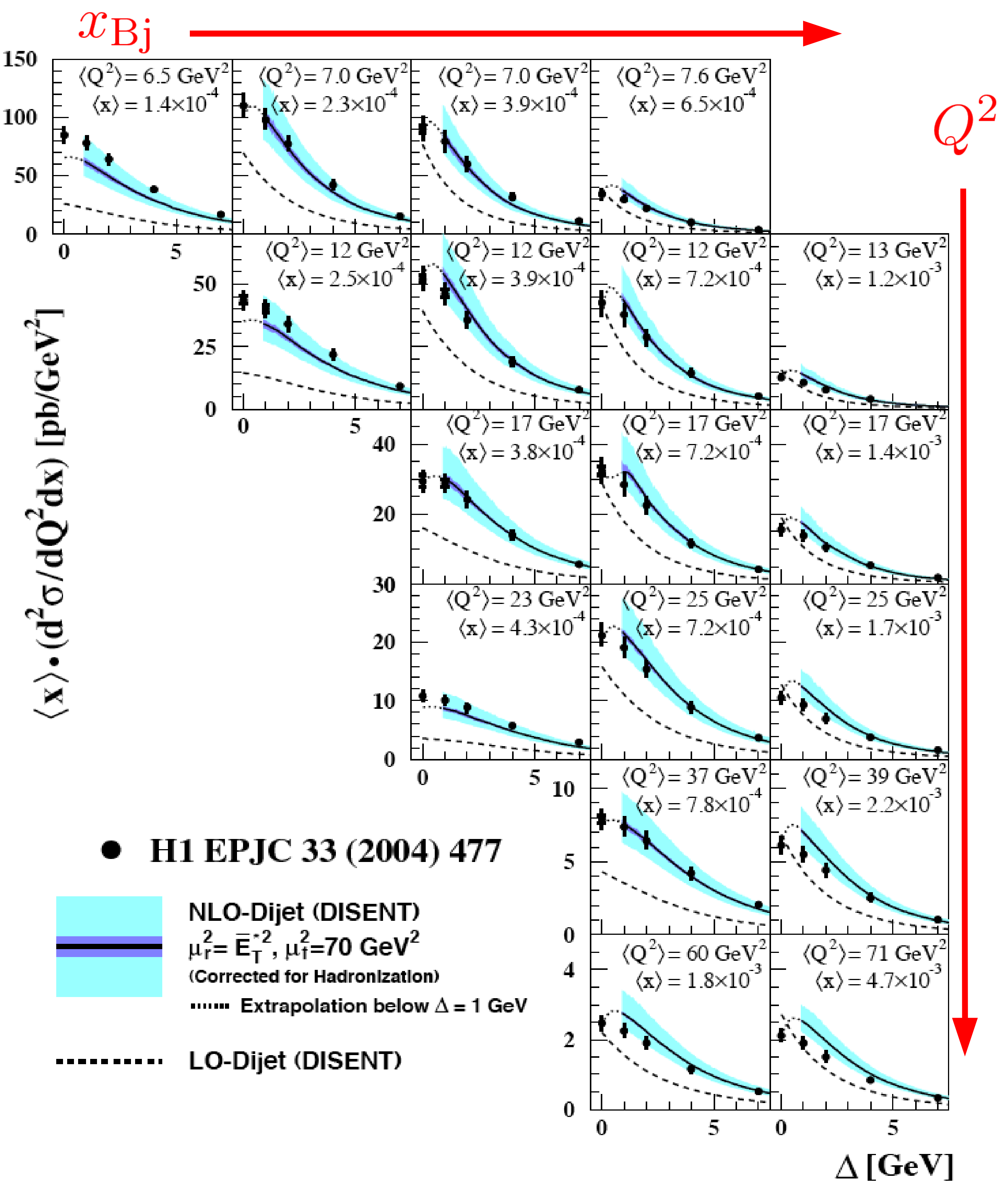
Fit to di-jet data

$$E_{\mathrm{T,\ jet\ 1}} > (5 + \Delta)\ \mathrm{GeV}$$

Total dijet cross-section as a function of Δ

- **NLO di-jet calculation fails in parts of phase space**

- **NLO di-jet calculation not possible for low Δ due to divergencies.**



Fit to di-jet data

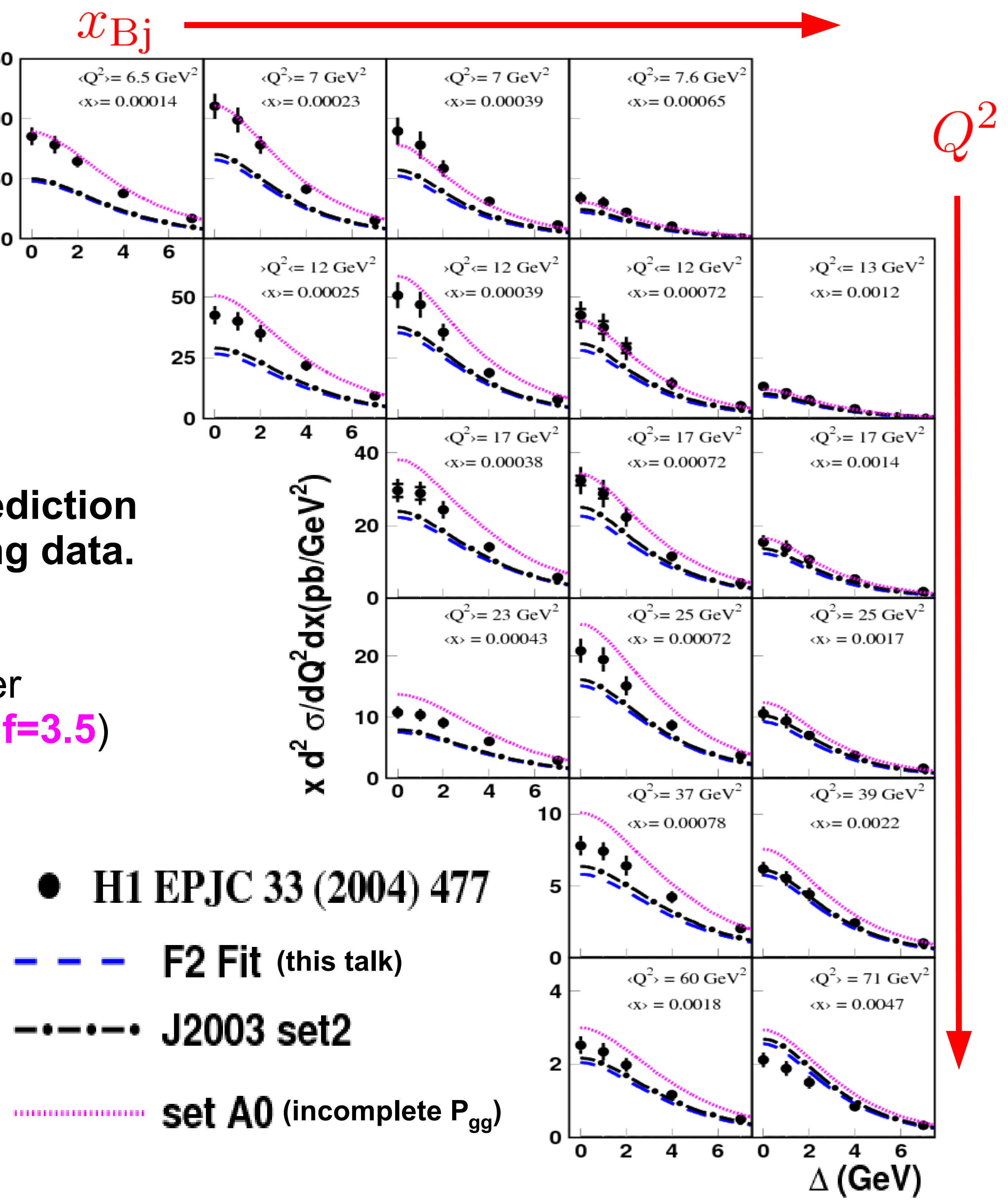
$E_{\mathrm{T,\ jet\ 1}} > (5 + \Delta)$ GeV

Total dijet cross-section as a function of Δ

Existing CCFM CASCADE prediction has some problems describing data.

Best is "**set A0**", one of the older uPDF for CASCADE. (**Chi2/ndf=3.5**)

(Compared to our new F2 fit set A0 does not include the complete CCFW splitting function, which we want.)



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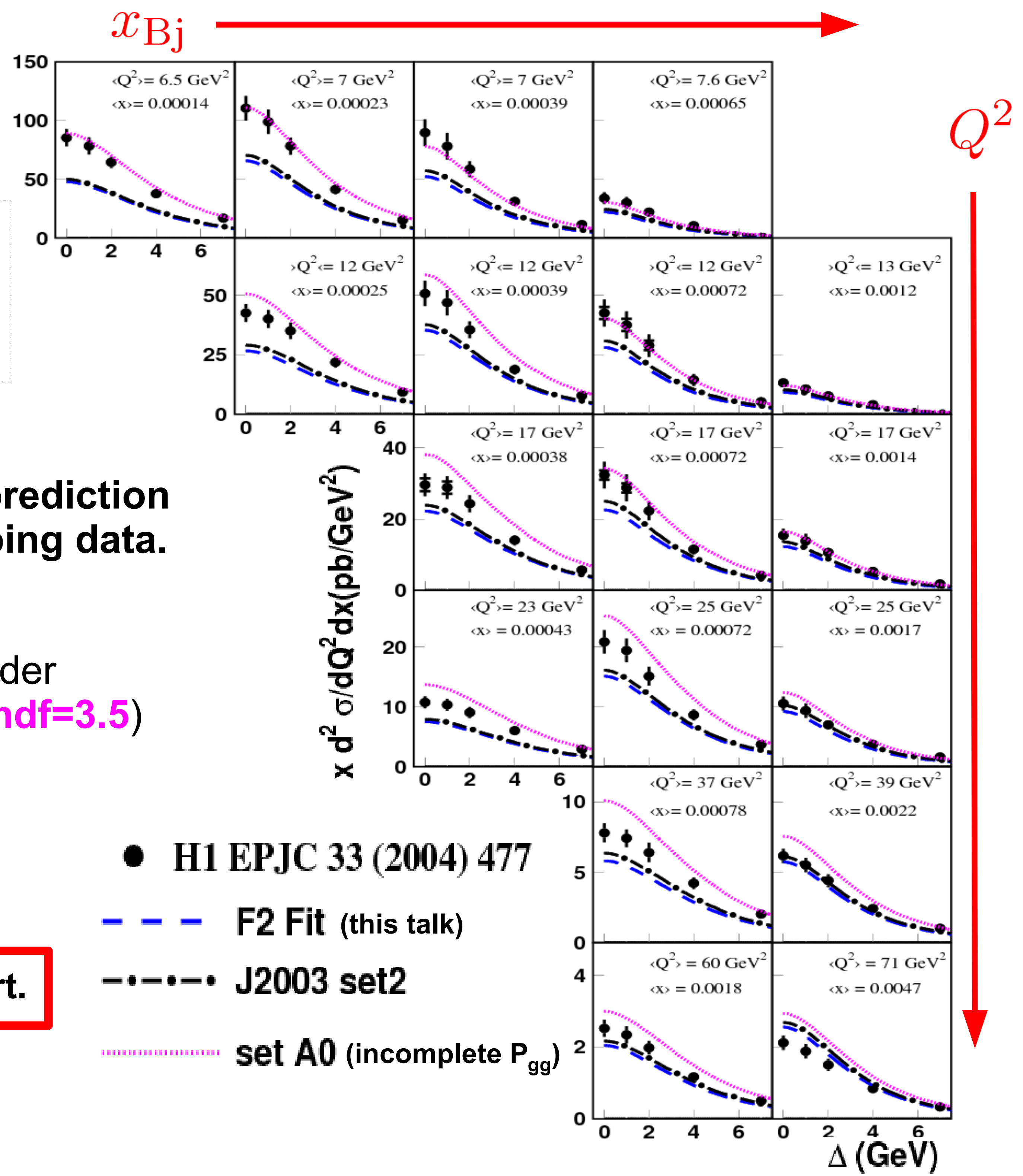
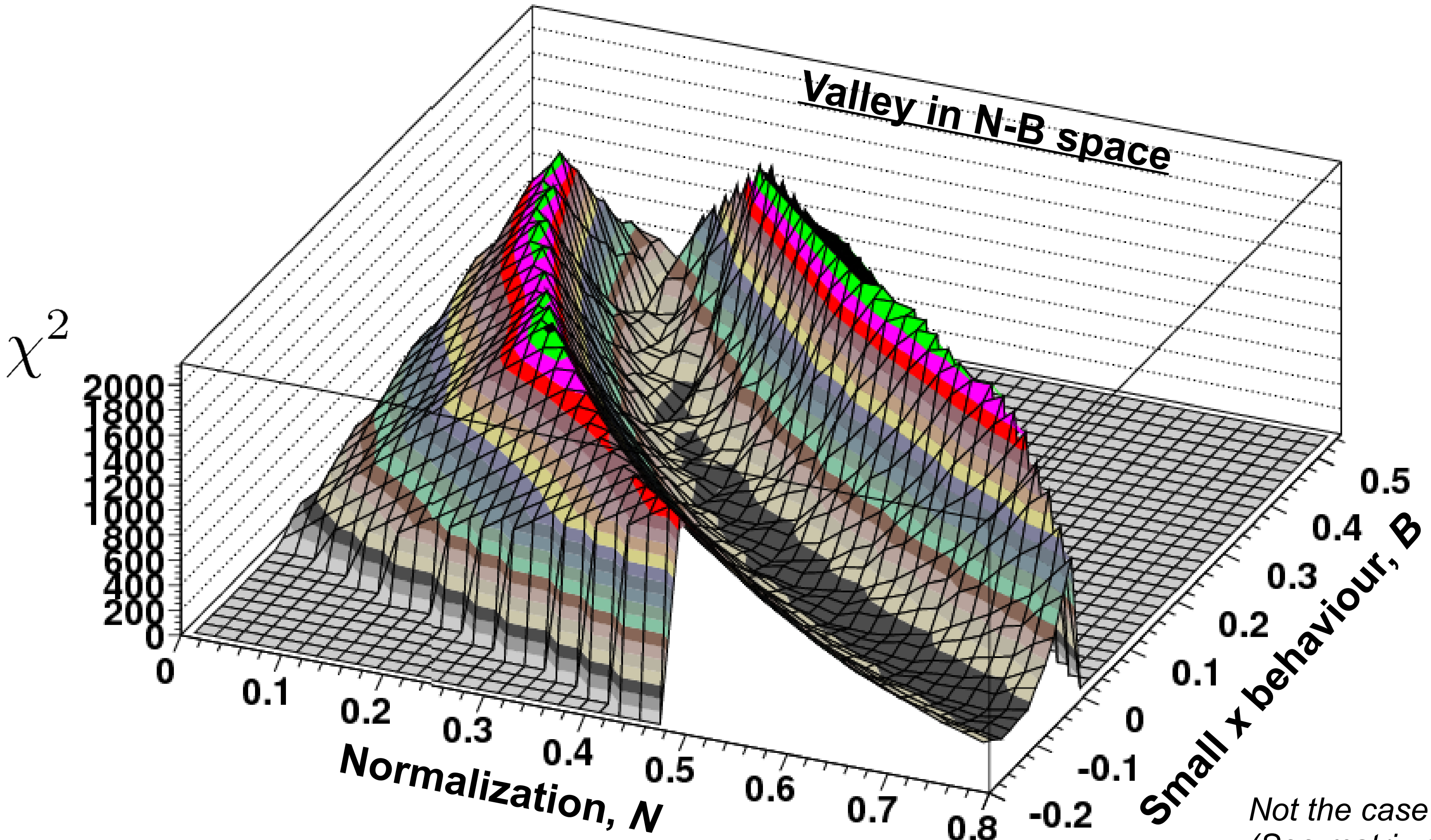
Remake fit – include the k_t-part.



Di-jet data insensitivity to the x behavior

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T-\mu)^2}{2\sigma^2})$$

The small *x* behaviour (**B)** is roughly arbitrary as long as we choose (fit) the correct normalization, N.

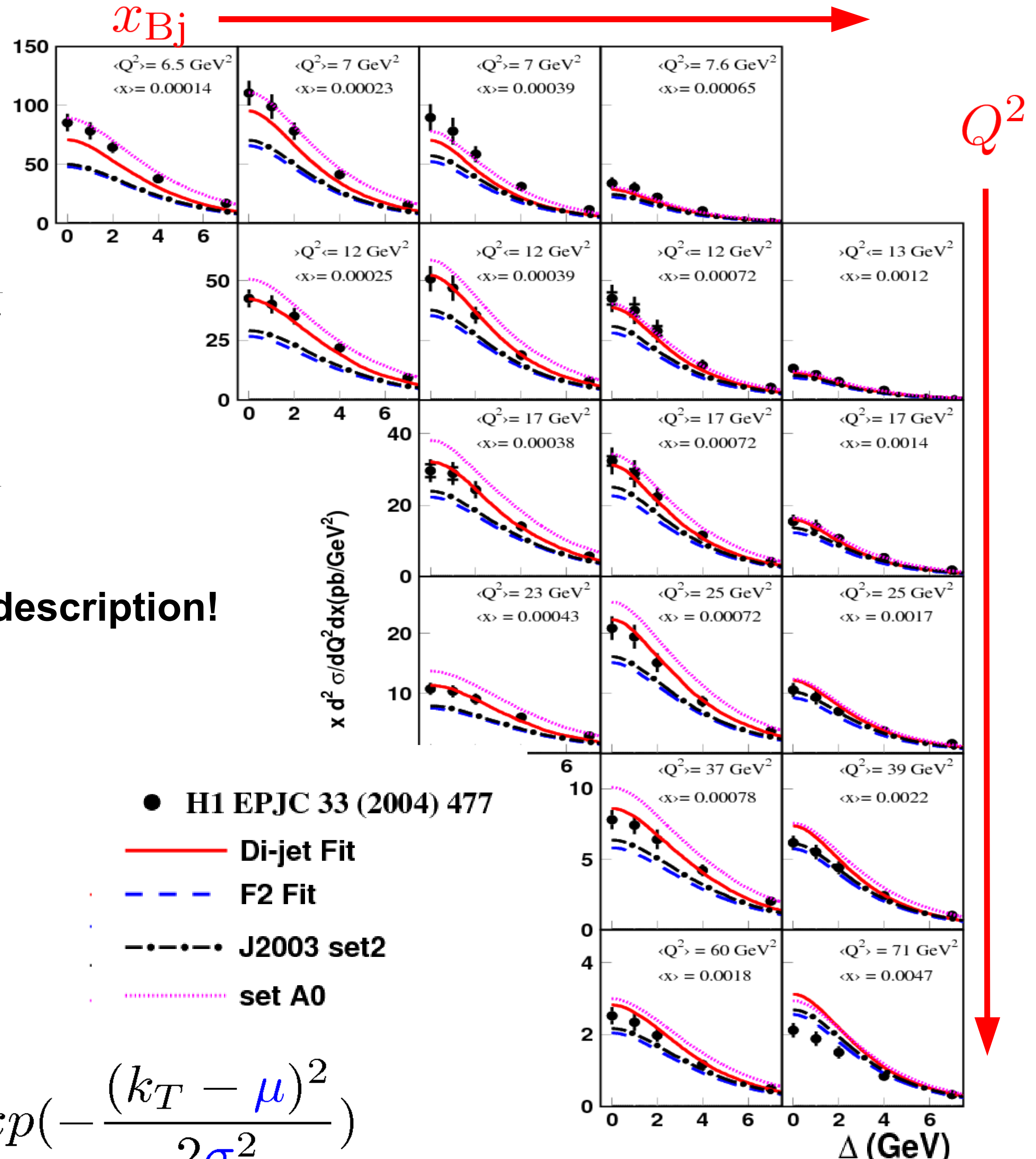


Not the case for F2.
(See matrix of 1-dim plots in backup)

Fit to di-jet data - results

$E_{\mathrm{T,\ jet\ 1}} > (5 + \Delta)\ \mathrm{GeV}$

Total dijet cross-section as a function of Δ



Fitted uPDF improves data description!

N	**0.28 +/- 0.02**
B	**0.25 +/- 0.03**
μ	**3.0 +/- 0.04**
σ	**2, fixed**
Chi2/ndf	**2.01**

$$N \cdot x^{-B} \cdot (1-x)^{C} \cdot exp(-\frac{(k_T - \mu)^2}{2\sigma^2})$$

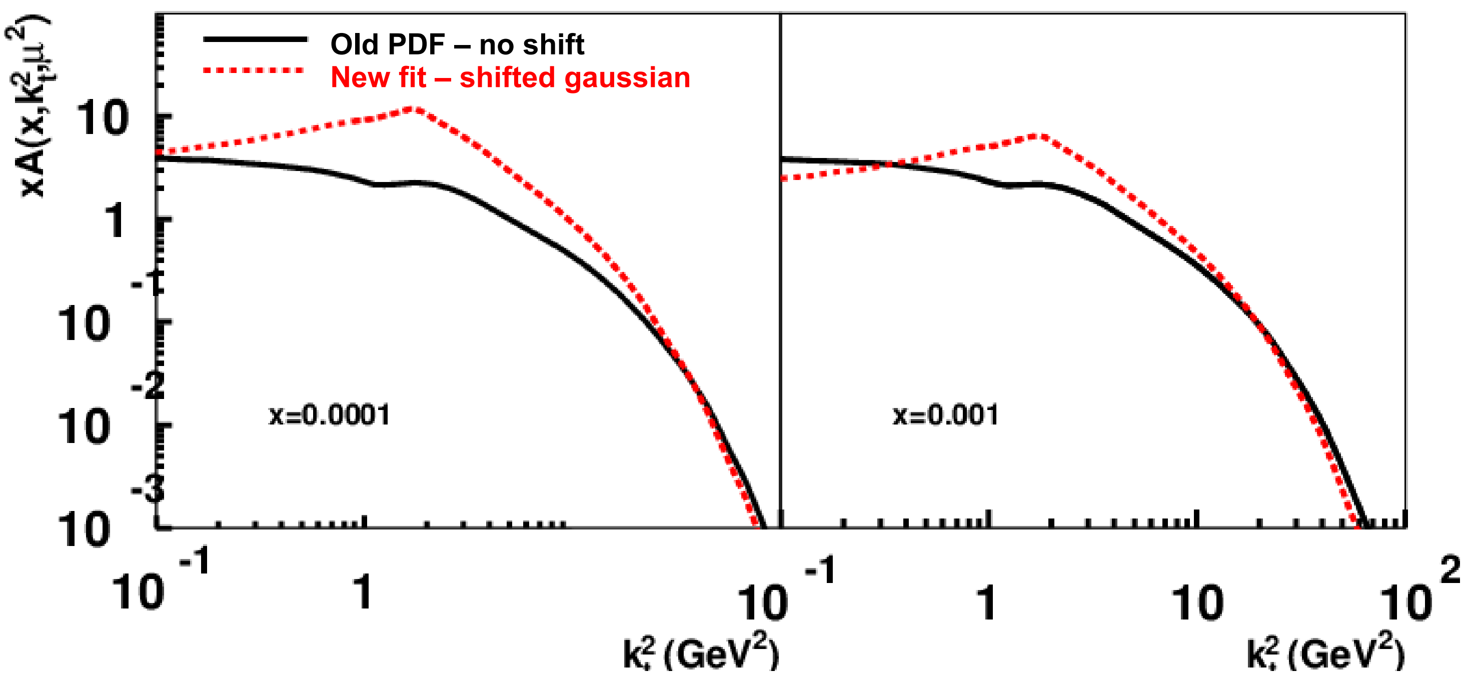
Comparison to existing uPDF

Former uPDF	
μ	0
Chi2/ndf	3.6

New Fit	
μ	3.0 +/- 0.04
Chi2/ndf	2

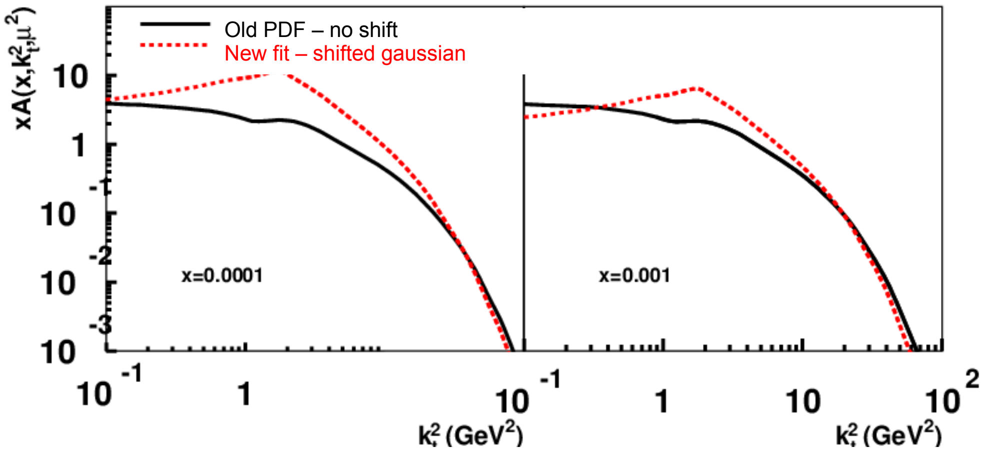
Set A0 starting distribution determined from fit to F2-data (H.Jung, Comp.Phys.Com. 143:100-111,2002)

The new fit to the dijet data suggest *stronger rising x* and a *shifted gaussian for k_t*.



What can we learn from the di-jets

- Sensitivity to the low k_t-region.



...but still we are in the perturbative region - require hard jets in the final state.
When measuring F2 at low Q2, there is risk that we leave the perturbative range.

What can we learn from the di-jets

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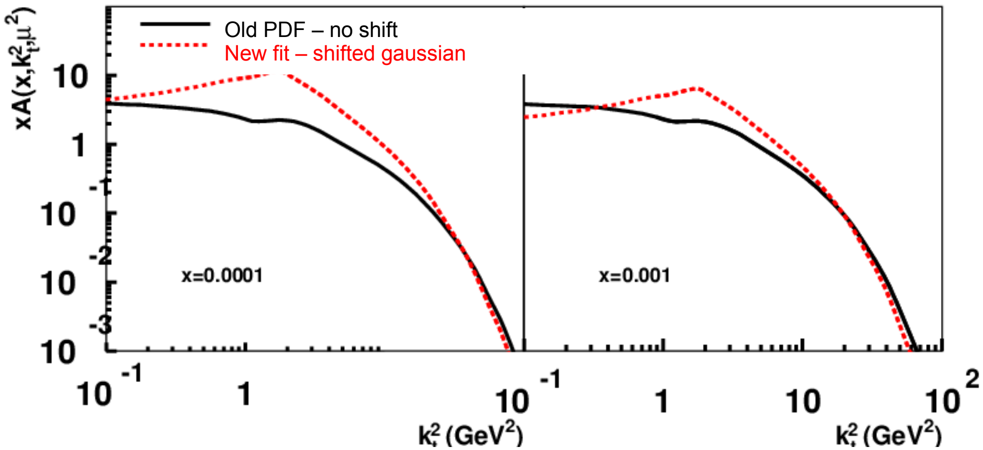


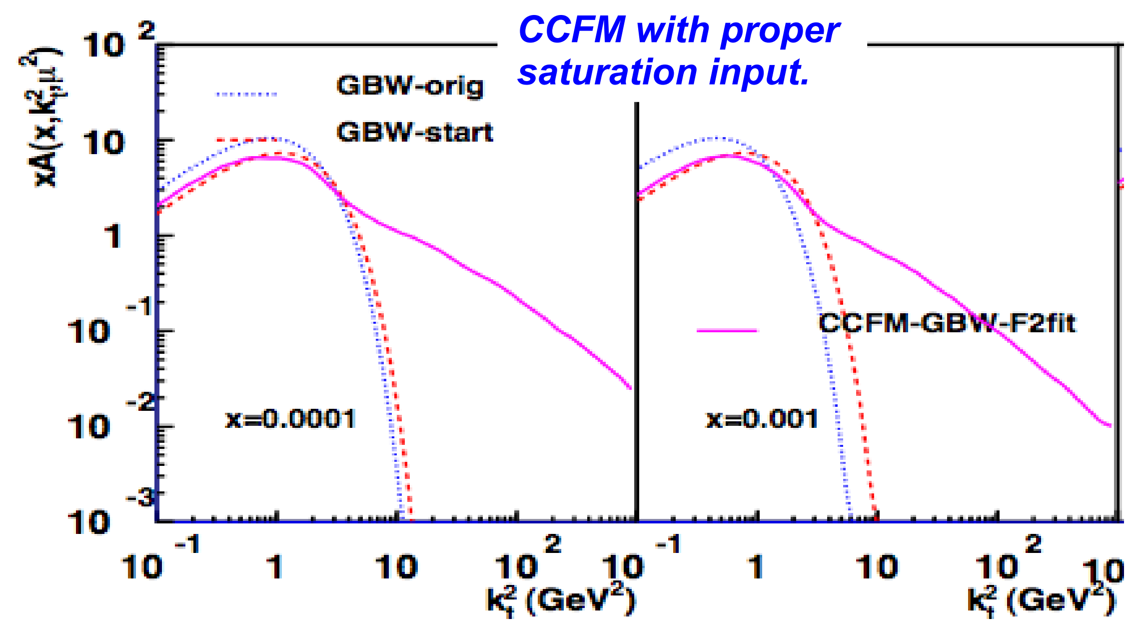
...but still we are in the perturbative region - require hard jets in the final state.
When measuring F2 at low Q2, there is risk that we leave the perturbative range.

- *Saturation effects mimicked?*

The suppression at low k_t is **also seen when using saturated PDFs**.

Saturation of parton density due to recombination of partons.

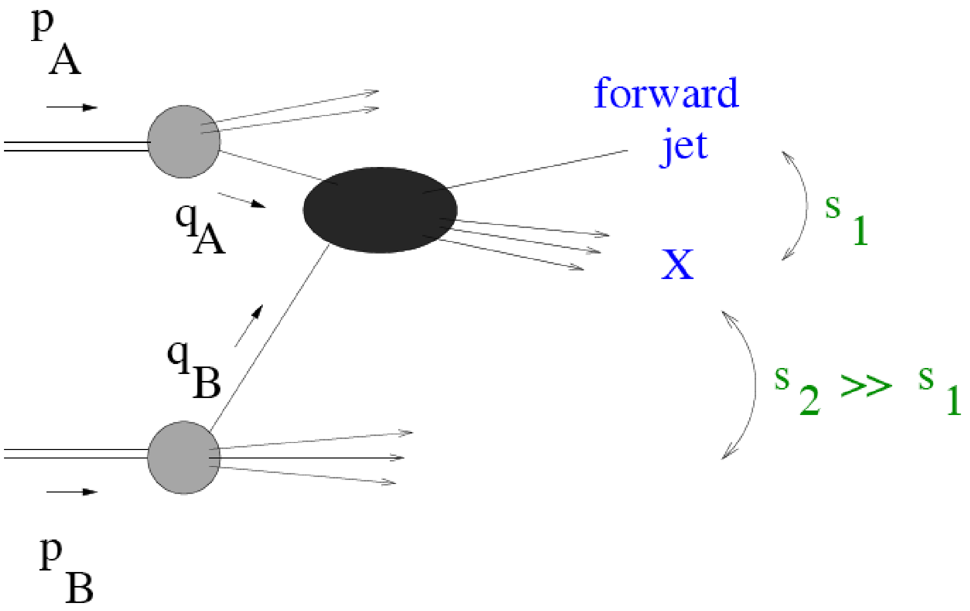
Jung, Kutak. arXiv 0812.4082



The contribution from quarks

•So far only unintegrated gluons (i.e. indirectly also sea quarks) in CASCADE.

•Valence quarks expected to be relevant for LHC. For example high Pt production:



Two scale process. With relevant physics for both $x \rightarrow 0$ and $x \rightarrow 1$. High sensitivity to parton dynamics.

Currently the k_t dependent quark PDF is taken from derivated CTEQ5.1.

Deak, Hautmann, Jung, Kutak. Published in JHEP within short.
arXiv:0908.0538

Summary

- Different data sensitive to different parts of the gluon.
- The x-dependent part of the uPDF is under control. Adding the extra factor (1-Dx) in the parameterization improves the description of F2 significantly.
- New interesting knowledge can be obtained from di-jets, suggests a suppressed gluon at low k_t. Similar effect as if using saturation approach.
- We successfully use a fast approach for fitting the uPDF. The method is based on grid interpolation in the uPDF parameter space.

albert.knutsson@desy.de

Back up

Back up

Back up

Back up

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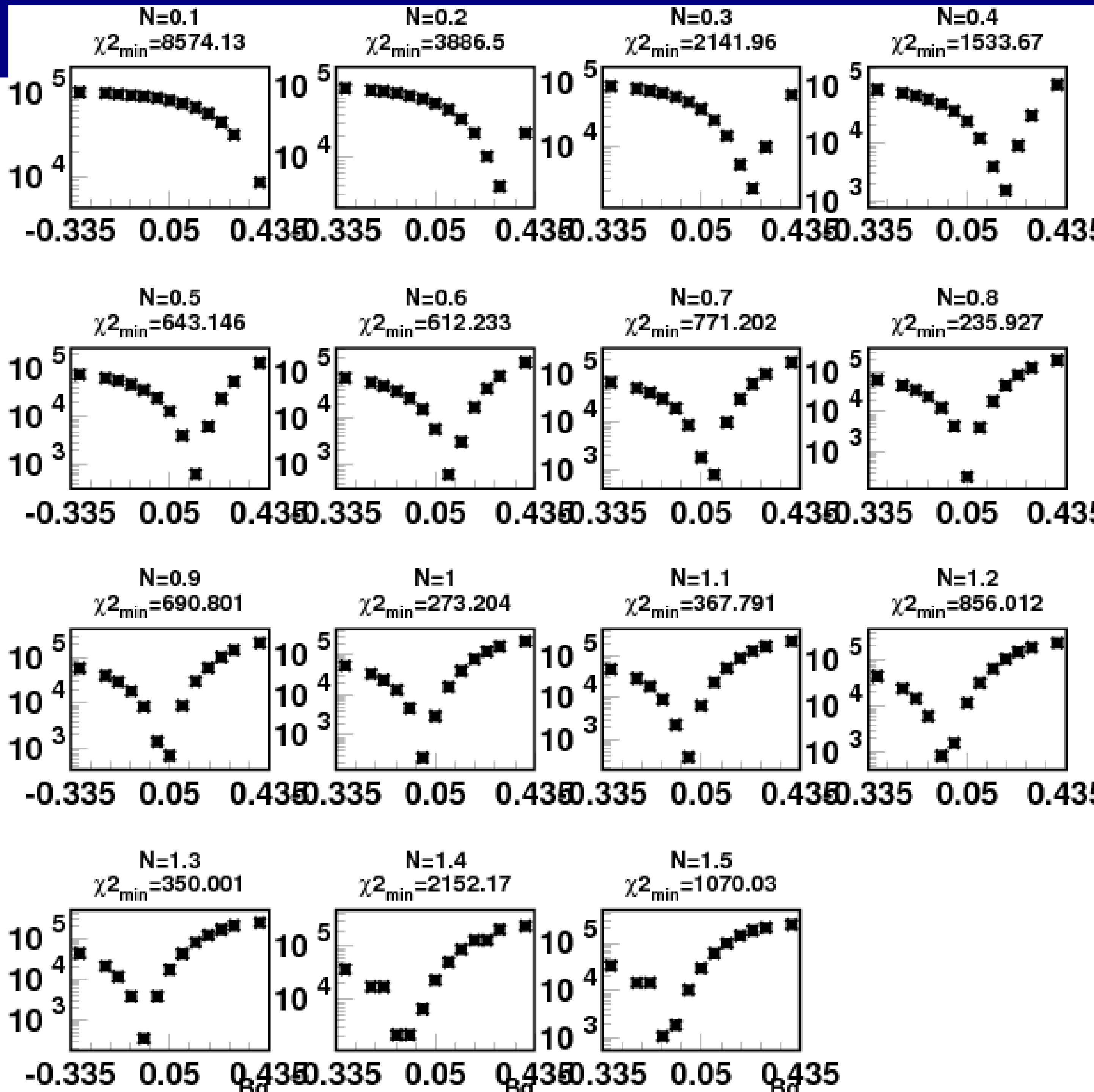
F2

Chi2 as a function of B, for different N values

Chi2_min written on top of each plot.

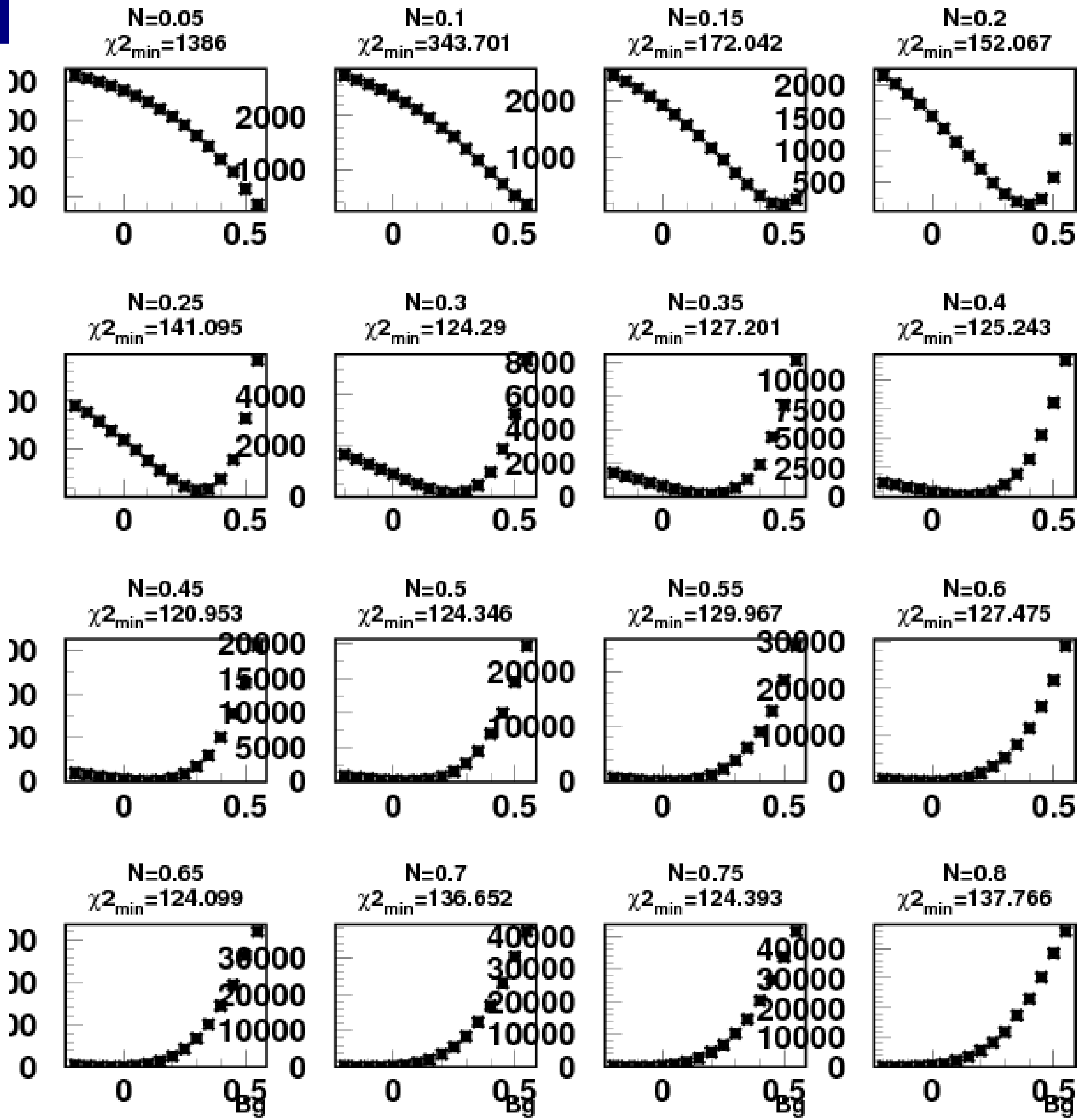
Minimum seems to be around N=0.8-1.0, B=0.0



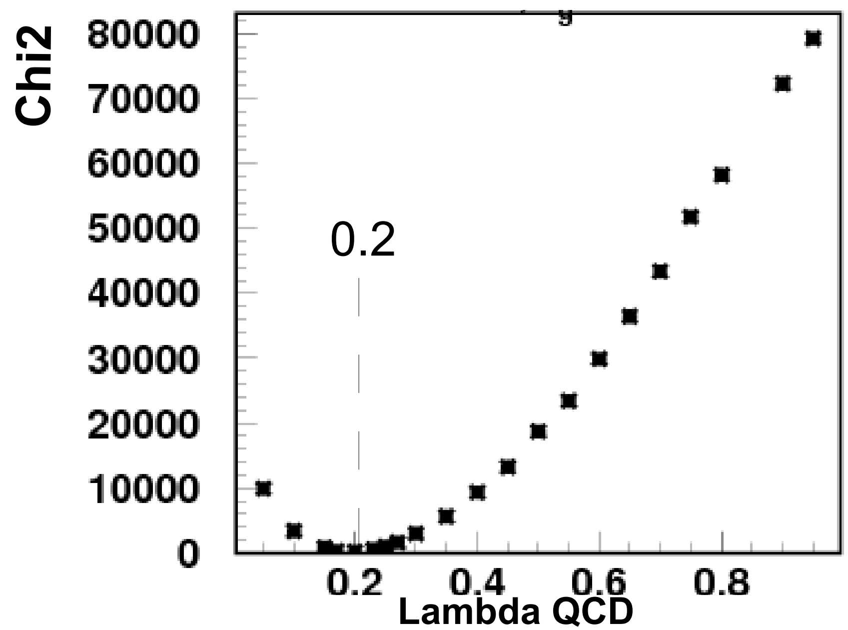
Chi2 as a function of Bg for different N values

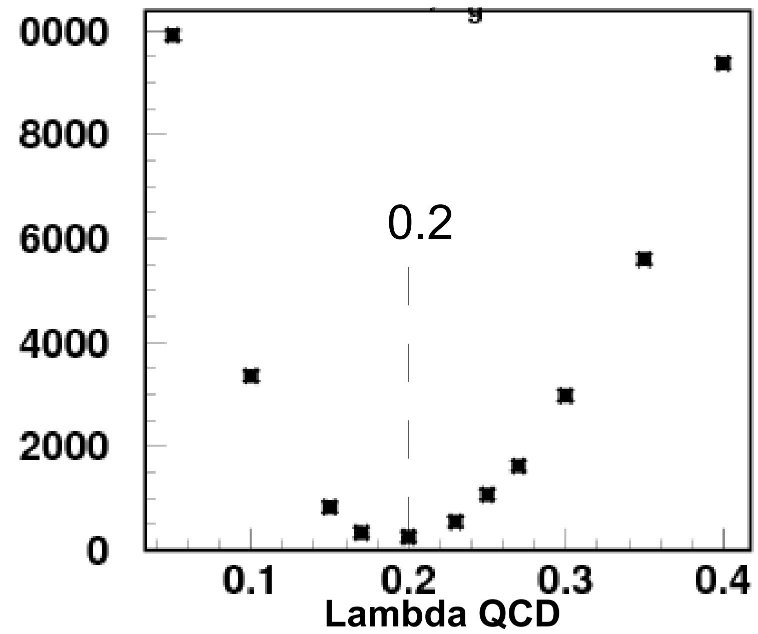
Dijets using Et and Eta cross-sections

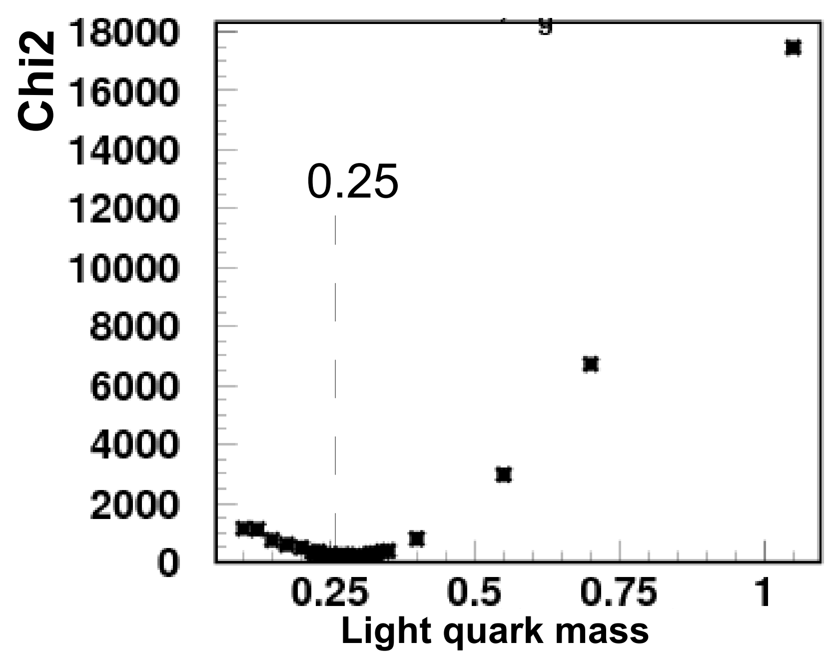
(data: DESY 03-160)

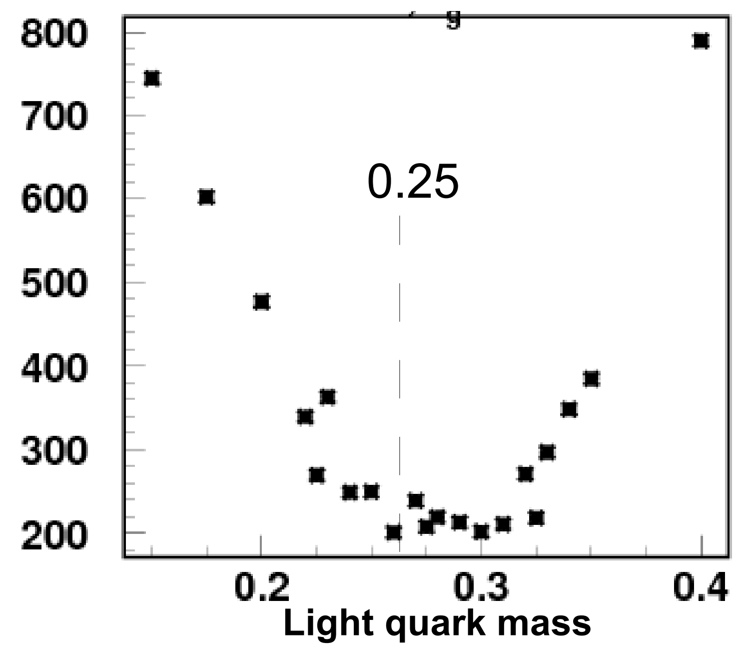


Scans


Chi2
80000
70000
60000
50000
40000
30000
20000
10000
0
0.2
0.2
0.4
0.6
0.8
Lambda QCD


0000
8000
6000
4000
2000
0
0.2
0.1
0.2
0.3
0.4
Lambda QCD


Chi2
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0.25
0.25
0.5
0.75
1
Light quark mass


800
700
600
500
400
300
200
0.25
0.2
0.3
0.4
Light quark mass

Error treatment

- The **statistical error of the MC** predictions are **propagated to the coefficients of the polynomial** fitted to the MC grid. Only the statistical errors of the MC enters, and thus a simple Chi2 calculation can be used:

$$\chi^2 = \Sigma \frac{(X_{MC} - X_{Polynomial})^2}{(\mathrm{Error}_{MC})^2}$$

- In the fit of the MC parameters to the experimental data the uncorrelated errors and the different correlated errors are treated separately according to:

$$\chi^2 = \Sigma \frac{(X_{Data} - X_{Polynomial})^2}{\alpha^2} - \Sigma_j \Sigma_{j'} B_j (A^{-1})_{jj'} B_{j'}$$

$\alpha^2 =$ Sum of uncorrelated errors (data and polynomial)

$\Sigma_j \Sigma_{j'} B_j (A^{-1})_{jj'} B_{j'} =$ Term related to the correlated systematic errors (vector *B*), and their correlations (matrix *A*)

(From the CTEQ group, hep/ph/0101051)

Summary Minimums

Fitting F2 in x<0.005, Q2>4.5 GeV. Using the usual PDF parameterization (N x^(-B))

N = 0.807 +/- 0.016
B = 0.029 +/- 0.004
Chi2/ndf = 1.2

Fitting the full F2, 1<Q2<150 GeV using the usual parameterization (N x^(-B))

N = 0.767 +/- 0.001
B = 0.028 +/- 0.000
Chi2/ndf = 5.4

Fitting F2 in x<0.005, Q2>4.5 GeV using (N x^(-B) (1-Dx))

N = 0.550 +/- 0.043
B = 0.082 +/- 0.011
D = -5.38 +/- 1.2
Chi2/ndf = 1.1

Fitting the full F2, 1<Q2<150 GeV using (N x^(-B) (1-Dx))

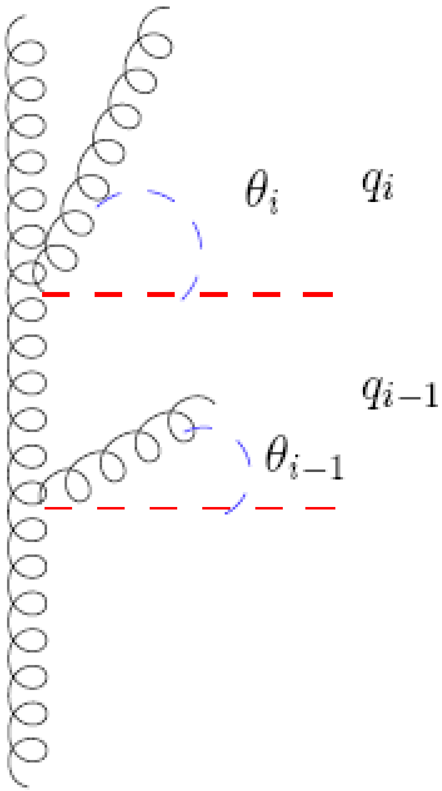
N = 0.487 +/- 0.007
B = 0.097 +/- 0.003
D = -5.10 +/- 0.35
Chi2/ndf = 2.8

The unintegrated gluon density

The uPDF starting distribution:

$$xA_0(x, k_T, \bar{q}_0) = N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T-\mu)^2}{2\sigma^2})$$

N: Normalization (fitted)
B: Small x behaviour (fitted)
C=4: Large x behaviour (kept fixed)
μ, σ Determines the shape of the intrinsic k_T of the gluon below k_T= 1.2 GeV (μ fitted)



Calculated at some starting scale ($\bar{q}_0$).
The uPDF is calculated for higher scales by emissions of gluons according to the *CCFM* evolution scheme.

The parameters N,B,C, μ ,σ, are not theoretically calculable.

→ We need to fit the uPDF to experimental data.

Acknowledgement

The new approach was developed for tuning Monte Carlo models

Suggested already 12 years ago...

“Tuning and test of fragmentation models based on identified particles and precision event shape data.”

Z.Phys.C73:11-60,1996

Also work on Tuning MC in Lund.

Parameter Optimisation in Monte Carlo Event Generators

Hendrik Hoeth

(University of **Lund**)

1st Mcnet School, IPPP Durham, 18-20th April 2007

We try to carry out the same method for fitting uPDFs.

Singular Value Decompostion

Number of Monte Carlo grid points > Coefficients → **Overdetermined system**

$$\sigma_{\text{poly}}(p_1, p_2) = A + B_1 p_1 + B_2 p_2 + C_1 p_1^2 + C_2 p_2^2 + C_3 p_1 p_2 + H.O.$$

i.e.

$$P_{n,m} X_m = \sigma_{n,poly}$$ **where**

$$X_n = (A, B_1, B_2, C_1, C_2, C_3, ...)$$
$$P_n = (1, p_1, p_2, p_1^2, p_1 p_2, p_2^2, ...)$$
$n =$ ***Grid point***

Approach based on SVD algorithm:

To obtain solution we minimize $|PX - \sigma|$

by χ^2 **-minimization**

SVD vs MINUIT

Could also use MINUIT, but it is sensitive on starting values.

	SVD	MINUIT	MINUIT bad starting values
Chi2 [Polynomial-MC]/ndf:	1.8	1.8	4.1

Minimization of polynomial coefficients stuck in local minimum

PROFFIT – PROgram For FITting

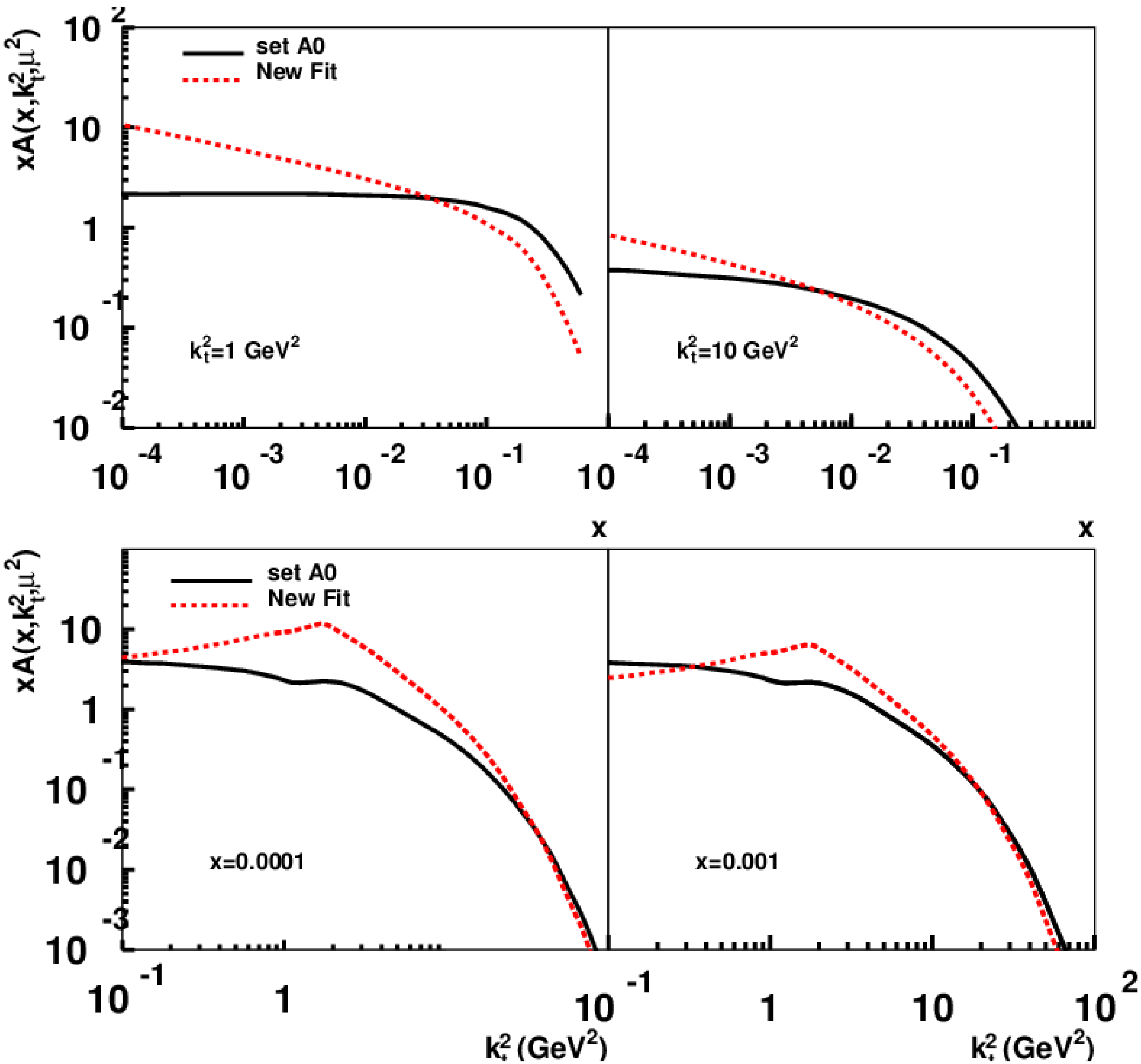
- **The method is implemented into a program – PROFFIT check for updates on www.hepforge.org/PROFFIT.**

- **A lot of data available for tuning in hztool**

  (*"HZTool is a library of routines which will allow you to reproduce an experimental result using the four-vector final state from Monte Carlo generators."*

  In the future replaced by RIVET)

Comparison to existing uPDF



$$N \cdot x^{-B} \cdot (1-x)^C \cdot exp(-\frac{(k_T-\mu)^2}{2\sigma^2})$$

Set A0

B	0
μ	0
Chi2/ndf	3.6

Set A0 starting distribution determined from fit to F2-data (H.Jung, Comp.Phys.Com. 143:100-111,2002)

New Fit

B	0.25 +/- 0.03
μ	3.0 +/- 0.04
Chi2/ndf	2

The new fit to the dijet data suggest *stronger rising x* and a *shifted gaussian for k_t*.

SVD vs MINUIT

Coefficients in 4th order polynomial determined from:

SVD	MINUIT	MINUIT bad starting values
-25404.9082	-25315.4624	358.56777
765676.064	762720.857	694969.672
357293.297	358067.861	-52198.7414
3091.77111	2347.15353	3582.44715
114841.02	140499.037	52826.8157
166905.85	157433.813	1929633.41
-31421.3098	-32900.9618	-61072.7505
-927589.152	-927572.803	98180.9293
-60480.3599	-61180.0691	-12538.2387
2524.9688	4162.80871	1618.16049
-1064150.37	-1135039.64	-465510.961
5799612.85	5804476.94	1334921.4
12981.5342	16228.0397	104971.842
2592311.1	2623536.1	1662889.02
313456.597	315635.922	284934.203
-26463.8328	-26091.529	-22961.2676
429940.571	419854.565	553178.07
318899.245	320294.755	-72213.1127
-23885.6361	-23727.0438	4989.53369
1446.24668	525.83837	308.886517
855372.625	918985.733	-135820.813
-3554618.	-3552083.15	354725.482
97974.8469	95580.1082	203313.617
-5838295.7	-5848044.35	536985.25
-214807.392	-216430.586	-978685.349
-9020.6301	-9326.67473	-22832.1865
10567702.9	10534823.1	2166665.32
-437402.716	-439016.175	427727.795

For example here, large difference between Coefficients.

Resulting in that MINUIT gets stuck in local minimum

Chi2 [Polynomial-MC]/ndf: 1.8 (SVD), 1.8 (MINUIT), 4.1 (MINUIT bad starting values)